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UNIVERSALITY SYSTEMS IN L^p, $1 \le p < 2$

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1. Introduction

Definition 1. A series of the form

$$\sum_{k=1}^{\infty} c_k \varphi_k(x), \tag{1}$$

where $\{\varphi_k(x)\}$, $x \in [0,1]$, is an orthonormed system (ONS) and c_k, $k \ge 1$, are real numbers, is said to be universal in $L^p_{[0,1]}$ (respectively, in $H = \bigcap\limits_{1 \le p < 2} L^p$) if, for any function $f(x) \in L^p_{[0,1]}$ ($f(x) \in H$, respectively) a sequence of increasing natural numbers $\{n_k\}$ exists such that a subsequence of partial sums $S_{n_k} = \sum\limits_{i=1}^{n_k} c_i \varphi_i(x)$ of series (1) converges to $f(x)$ in the metric of $L^p_{[0,1]}$ (in all metrics of $L^p_{[0,1]}$, $1 \le p < 2$, respectively).

Definition 2. An ONS $\{\varphi_n(x)\}$ will be called a universality system in $L^p_{[0,1]}$ (in H, respectively) if along this system a series of the form (1) exists which is universal in $L^p_{[0,1]}$ (in H, respectively).

Let us note at once that a trigonometric system is not a universality system in any $L^p_{[0,1]}$, $p \ge 1$. But, nevertheless, it turns out that an ONS $\{\varphi_n(x)\}$ exists which is a universality system in both H and any fixed $L^p_{[0,1]}$, $1 \le p < 2$.

We should note that first universal (in the sense of the convergence almost everywhere (a. e.)) trigonometric series were constructed by D.Ye. Men'shov (see [1]) and V.Ya. Kozlov (see [2]). Namely, they constructed series of the form

$$\frac{1}{2} + \sum_{k=1}^{\infty} a_k \cos kx + b_k \sin kx \tag{2}$$

such that for any measurable on $[0, 2\pi]$ function $f(x)$ a sequence of integers $1 < n_1 < n_2 < \cdots < n_k < \cdots$ can be found such that

$$\lim_{k \to \infty} S_{n_k}(x) = f(x) \ \text{ a. e. on } \ [0, 2\pi]$$

($S_n(x)$ is the partial sum of order n of series (2)). This result was extended in [3] onto arbitrary complete ONS. In this article we obtain a necessary and sufficient condition on an ONS $\{\varphi_n(x)\}$ in order for this be a universality system in $L^p_{[0,1]}$, $1 \le p < 2$.