Russian Mathematics (Iz. VUZ)
Vol. 48, No. 11, pp. 79–83, 2004

Izvestiya VUZ. Matematika
UDC 514.763

THREE-WEBS DEFINED BY RICCATI EQUATION

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Introduction. In [1] we started to study systematically special classes of curvilinear three-webs which are determined by simple relations on the relative web invariants. We proved that one of the covariant derivatives of the web curvature b vanishes if and only if two of the web curve families are determined by the generalized Abel equations

$$\frac{dy}{dx} = -ay^n + b(x)y^{n-2} + \cdots + c(x).$$

In [2] we analytically characterized curvilinear three-webs determined by an arbitrary first order linear differential equation. This class can be also characterized by the following relations on the covariant derivatives of the web curvature:

$$bb_{22} - (b_2)^2 = 0, \quad bb_{21} - b_1 b_2 - b^3 = 0.$$

In the present paper we consider a three-web W determined by the Riccati equation

$$y' = f(x)y^2 + g(x)y + h(x). \tag{1}$$

This three-web consists of a Cartesian net and the family of integral curves of (1). Our main result is

Theorem. *Invariants of a curvilinear three-web satisfy the equation*

$$b_{222}b - b_{22}b_2 = 0. \tag{2}$$

if and only if this three-web is determined by the Riccati equation.

Recall that three-webs are considered up to local diffeomorphisms. This means that the parameters of foliations of the web admit smooth monotone transformations. In other words, (1) is defined up to transformations

$$x = \alpha(\widetilde{x}), \quad y = \alpha(\widetilde{y}), \tag{3}$$

moreover, the same transformations can be applied to the integration constant, the parameter of third foliation of the web.

1. Let us define the foliations λ_α of a web W by the Pfaff equations $\omega_1 = 0$, $\omega_2 = 0$, $\omega_1 + \omega_2 = 0$ (see [3]). The basic forms ω_1 and ω_2 satisfy the structure equations

$$d\omega_1 = \omega_1 \wedge \omega, \quad d\omega_2 = \omega_2 \wedge \omega \tag{4}$$

and

$$d\omega = b\omega_1 \wedge \omega_2, \tag{5}$$

where b is called the curvature of the three-web W.