Statistical Characteristics of Continuous Functions and Statistically Weakly Invariant Sets of Controllable System

Ya. Yu. Larina* and L. I. Rodina**

Udmurt State University
ul. Universitetskaya 1, Izhevsk, 426034 Russia
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Abstract—We continue the investigation of expansion of a concept of invariance for sets which consists in studying statistically invariant sets with respect to control systems and differential inclusions. We consider the statistical characteristics of continuous functions: Upper and lower relative frequency of containing for graph of a function in a given set. We obtain conditions under which statistical characteristics of two various asymptotical equivalent functions coincide; then by the value of one of them it is possible to calculate the value of another one. We adduce the equality for finding relative frequencies of hitting functions the given set in the case when the distance from the graph of one of functions to the given set is a periodic function. A consequence of these statements are conditions of statistically weak invariance of a set with respect to controlled system. For some almost periodic functions we obtain the formulas by which we can calculate the mean values and the statistical characteristics. We also consider the following problem. Let the number $\lambda_0 \in [0, 1]$ be given. It is necessary to find the value $c(\lambda_0)$ such that the upper solution $z(t)$ of the Cauchy problem does not exceed $c(\lambda_0)$ with the relative frequency being equal λ_0. Depending on statement of the problem, a value $z(t)$ can be interpreted as the size of population, energy of a particle, concentration of substance, size of manufacture or the price of production.

INTRODUCTION

The research problem of statistical characteristic of attainability set arises when we study invariant and statistically invariant sets of controllable system and differential inclusions. There is a new approach for enlargement of invariance definition in the works [1−5] devoted to this subjects. This approach is conditioned by many applied tasks in economics, ecology, engineering and lies in investigation of relative frequency freq(X) of set of attainability of controllable system in a given set $\mathfrak{M}$. If such frequency equals one, then $\mathfrak{M}$ is called statistically invariant. In the works [1−5] in terms of Lyapunov functions and Clark derivative the conditions are obrained such that frequency freq(X) may be estimated by the following characteristic

$$\varkappa = \lim_{\vartheta \to \infty} \frac{\mathrm{mes}\{t \in [0, \vartheta] : z(t) \leq 0\}}{\vartheta},$$

where the function $z(t)$ is upper solution of scalar Cauchy problem. In present paper we investigate conditions of statistical weakly invariance of set $\mathfrak{M}$, find mean values and statistical characteristics for almost periodic functions in the Bohr sense.

Special attention is paid to the tasks where different statistical characteristics are calculated or are estimated. In particular, the following problem is examined. Let a number $\lambda_0 \in [0, 1]$ be given. It is necessary to find value $c(\lambda_0)$ such that upper solution $z(t)$ of Cauchy problem is less than or equals $c(\lambda_0)$ with relative frequency being equal λ_0. Solution $z(t)$ may be interpreted as the size of population, energy of particle, concentration of substance, quantity or price of production.

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*E-mail: yana_larina89@mail.ru.
**E-mail: box0589@udmnet.ru.