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Three Classes of Weitzenböck Manifolds

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Abstract—A Weitzenböck manifold is a triplet defined by a differentiable manifold with a metric g of certain signature and a linear connection with zero curvature tensor and nonzero torsion tensor which is a metric connection with respect to g. The theory of such manifolds is called the "new theory of gravity". We study properties of three classes of Weitzenböck manifolds and prove some vanishing thorems.

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1. INTRODUCTION

1.1. Consider an n-dimensional ($n \geq 0$) C^∞ manifold M with linear connection $\overline{\nabla}$ having zero curvature $\overline{R} = 0$ and nonzero torsion $\overline{S} \neq 0$. In this case ([1], pp. 133–135), any vector X_x at a point $x \in M$ can be included into an absolutely parallel vector field X which is (locally) uniquely defined. For this reason, the connection $\overline{\nabla}$ is said to be a connection of absolute parallelism.

E. Cartan has constructed [2] a similar connection on the 2-dimensional sphere without poles in the Euclidean space. This connection has the additional metricity property: two absolutely parallel vector fields X_1 and X_2 are orthogonal.

1.2. More than fifty years later, E. Cartan's idea was developed in a series of papers devoted to the so-called "new theory of gravity" [3] of a space-time (M, g) which, in addition to the Levi-Civita connection ∇, has a connection $\overline{\nabla}$ with nonzero curvature $\overline{R}$ and torsion $\overline{S}$ such that $\overline{\nabla} g = 0$ (metricity property).

In particular, space-times with zero curvature $\overline{R} = 0$ and nonzero torsion $\overline{S} \neq 0$ were studied. These spaces are referred to as "Weitzenböck spaces (manifolds)" or "teleparallelism spaces" (see, e.g., [4–6]).

In [7] and [8], a classification of manifolds $(M, g, \overline{\nabla})$ has been obtained on the base of pointwise irreducible decomposition $T = T_1 + T_2 + T_3$ of the deformation tensor $T = \overline{\nabla} - \nabla$ with respect to the action of the group $O(q)$, where $q = g_x$ for an arbitrary point $x \in M$. In the next section, we will study properties of the three classes of Weitzenböck manifolds defined by the relations $T = T_1$, $T = T_2$, and $T = T_3$.

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