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A DIFFERENTIATION OPERATOR WITH NONREGULAR BOUNDARY CONDITIONS

Yu. T. Sil'chenko

1. Consider a differential expression $lu = (-1)^m u^{(2m)}(x)$, $x \in [0,1]$, and a system of boundary conditions

$$L_\nu u = d_\nu u^{(k_\nu)}(0) + \beta_\nu u^{(k_\nu)}(1) + T_\nu u = 0, \tag{1}$$

where α_ν, β_ν are complex numbers, $|\alpha_\nu| + |\beta_\nu| \neq 0$, $0 \le k_1 \le \dots \le k_\nu \le k_{\nu+1} \le \dots \le 2m-1$, T_ν are bounded linear functionals in $W_p^{k_\nu - 1}(0,1)$ for some $p \in [1,\infty)$, $\nu = 1,2,\dots,2m$.

We denote by $\omega_1, \omega_2, \dots, \omega_{2m}$ all the $2m$-th roots of 1. The boundary conditions (1) are called *regular* (see [1], [2]) if the determinant

$$\begin{vmatrix} \alpha_1\omega_1^{k_1} \dots \alpha_1\omega_m^{k_1} & \beta_1\omega_{m+1}^{k_1} \dots \beta_1\omega_{2m}^{k_1} \\ \dots & \dots \\ \alpha_{2m}\omega_1^{k_{2m}} \dots \alpha_{2m}\omega_m^{k_{2m}} & \beta_{2m}\omega_{m+1}^{k_{2m}} \dots \beta_{2m}\omega_{2m}^{k_{2m}} \end{vmatrix} \tag{2}$$

differs from zero.

We introduce in the space $L_p = L_p(0,1)$ an operator $Lu = lu$ with the domain $D(L)$ consisting of functions from the class $W_p^{2m}(0,1)$, which satisfy the boundary conditions (1). Then (see [1], [2]) the operator L for $|\lambda| \ge R_\varepsilon > 0$, $|\arg\lambda| < \pi - \varepsilon$ has the resolvent $(L+\lambda I)^{-1}$, satisfying the estimate $\|(L+\lambda I)^{-1}\| \le C|\lambda|^{-1}$ for a certain $\varepsilon > 0$ ($\varepsilon < \pi$). If conditions (1) are not regular, then the resolvent may be not of maximal order of decrease and even may increase as $|\lambda| \to \infty$. Thus, if $lu = -u''(x)$, $L_1 u = u(0)$, $L_2 u = \int_0^1 u(x)dx$, then $\|(L+\lambda I)^{-1}\| \le C|\lambda|^{-\frac{1}{2}-\frac{1}{2p}}$. If $L_1 u = u(0)$, $L_2 u = u'(0)$, then $\|(L+\lambda I)^{-1}\| = C|\lambda^{-1}e^{\sqrt{\lambda}}|$.

In the present article we separate out some classes of nonregular boundary conditions, for which the resolvent has a decreasing norm.

2. Consider boundary conditions (1) for $\nu = 1,2,\dots,2m-r$ $(0 \le r \le 2m)$, while the rest r conditions are supposed to be given via the formulas

$$L_\nu u = T_\nu u = 0 \quad (\nu = 2m-r+1,\dots,2m), \tag{3}$$

where $T_{2m-r+k}u = \int_0^1 \varphi_k(x)u(x)dx$ $(k = 1,2\dots,r)$, and $\{\varphi_k(x)\}$ is a given system of continuous and linearly independent functions on $[0,1]$. Assume that for these functions there hold

$$\begin{aligned} &\int_0^1 \varphi_k(x)\exp(\rho\omega_j x)dx = C_{kj}\rho^{-n_k} + a_{kj}(\rho)\rho^{-n_k} \quad (j = 1,2,\dots,m), \\ &\int_0^1 \varphi_k(x)\exp[\rho\omega_j(x-1)]dx = C_{kj}\rho^{-n_k} + a_{kj}(\rho)\rho^{-n_k} \quad (j = m+1,\dots,2m); \end{aligned} \tag{4}$$