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Boundary Theorems of Uniqueness for Logarithmic-Subharmonic Functions

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Abstract—We investigate the boundary theorems of uniqueness for certain important classes of logarithmic-subharmonic functions defined on the unit disk.

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Numerous important works of famous mathematicians are dealing with uniqueness theorems for meromorphic functions in the unit disk D. A reader can find a detailed description of these results in [1–6]. Certain results of that kind are extended on subharmonic functions; see, e.g., [7–10]. The author continues studies of this subject for logarithmic-subharmonic functions. A non-negative subharmonic functions $u(z)$ is called logarithmic-subharmonic if $\ln u(z)$ is also subharmonic function. We keep notation and definitions of papers [11, 12]. In addition, we say that a subset S_0 of the disk D satisfies condition (C) (see [6]) if

1) the set $E = \{|z|; z \in S_0\}$ is dense in certain interval $[r_0, 1)$ of the real axis,

2) for any $\eta > 0$ there exists a value $\delta > 0$ such that $|\arg z| < \eta$ for all $z \in S_0$ lying inside the ring $1 - \delta < |z| < 1$.

We denote by S_ξ the image of S_0 under rotation $z' = \xi z$, $|\xi| = 1$, then S_ξ has on Γ a unique limit point ξ. A set $N \subset \Gamma$ is called metrically dense on certain arc $\gamma \subset \Gamma$, if linear measure $\mathrm{mes}(\gamma' \cap N)$ is positive for any arc $\gamma' \subset \gamma$. We say (see [13]) that $u(z)$ is subordinated in D to subharmonic function $v(z)$ if $u(z) = v[\omega(z)]$, where the function $\omega(z)$ is analytic in D, and $\omega(0) = 0$, $|\omega(z)| < 1$. It is known (see ([13], P. 109) that the function $u(z)$ is subharmonic in D. The concept of subordination can be introduced analogously in the case of analytic $v(z)$. Then subordinated function $u(z)$ is also analytic in unit disk D. A point $\xi \in \Gamma$ is called uncertainty point if there exist two paths j_1 and j_2 ending at the point ξ such that $C_{j_1}(f, \xi) \cap C_{j_2}(f, \xi) = \emptyset$. We denote by $I(\alpha, \beta)$ the arc on Γ with end points $e^{i\alpha}$ and $e^{i\beta}$, where $0 \le \alpha < \beta \le 2\pi$. Assume that $\sigma(I) = D \cap N_\delta(\xi)$, where $\delta > 0$ and $N_\delta(\xi)$ is δ-neighborhood of the point $\xi = e^{i\theta} \in \Gamma$. The boundary of domain $\sigma(I)$ on Γ is the arc $I(\theta - \delta, \theta + \delta)$. Let $S(\alpha, \beta) = \{z = re^{i\theta} : \alpha < \theta < \beta, 0 \le r < 1\}$ be a sector of the disk D. We call a set $E \subset \Gamma$ (see [2]) the set of the first category if it is a union of countable family of nowhere dense sets. Otherwise it is the set of the second category. A set $E \subset I(\alpha, \beta)$ is called remainder set, if its complement in $I(\alpha, \beta)$ is a set of the first category. We say that a subharmonic function $u(z)$ has a harmonic majorant in the domain G, if there exists a harmonic function $v(z)$ such that $u(z) \le v(z)$ in G. We put $R_+ = [0, +\infty]$.

We consider two theorems, which were obtained by M. Arsove, in a convenient for our consideration form. Theorem A* is a generalization of classic Littlewood theorem for subharmonic in D functions (see [13]), and Theorem B* is a subharmonic analog of the N. N. Luzin and I. I. Privalov uniqueness theorems [1].

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