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First Nonzero Eigenvalue of a Pseudo-umbilical Hypersurface in the Unit Sphere

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Abstract—S. Deshmukh has obtained interesting results for first nonzero eigenvalue of a minimal hypersurface in the unit sphere. In the present article, we generalize these results to pseudo-umbilical hypersurface and prove: What conditions are satisfied by the first nonzero eigenvalue λ_1 of the Laplacian operator on a compact immersed pseudo-umbilical hypersurface M in the unit sphere S^{n+1}. We also show that a compact immersed pseudo-umbilical hypersurface of the unit sphere S^{n+1} with $\lambda_1 = n$ is either isometric to the sphere S^n or for this hypersurface an inequaluity is fulfilled in which sectional curvatures of the hypersuface M participate.

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1. INTRODUCTION

Let S^{n+1} be an $(n+1)$ dimensional unit sphere of constant curvature 1 and M be compact n-dimensional hypersurface isometrically immersed in S^{n+1}. Let h be the second fundamental form of the immersion and H be the mean curvature vector. Denote by $\langle \cdot, \cdot \rangle$ the scalar product of S^{n+1}. If there exists a function λ on M such that

$$\langle h(X,Y), H\rangle = \lambda \langle X, Y\rangle$$

for any tangent vectors X and Y on M, then M is called a pseudo-umbilical hypersurface of S^{n+1} [1]. It is clear that $\lambda \geq 0$. If the mean curvature vector $H = 0$ identically, then M is called a minimal hypersurface of S^{n+1}. Every minimal hypersurface of S^{n+1} is itself a pseudo-umbilical hypersurface.

There are several results on the first nonzero eigenvalue λ_1 of the Laplacian operator of a minimal hypersurface in the unit sphere [2–4].

However, recently S. Deshmukh [4] studied immersed compact minimal hypersurfaces of the unit sphere S^{n+1} and proved two following statements.

Theorem A. Let M be an immersed compact minimal hypersurface of the unit sphere S^{n+1}. Then the first nonzero eigenvalue λ_1 of the Laplacian operator on M satisfies one of the following:

(i) $\lambda_1 = n$,

(ii) $\lambda_1 \geq n + \frac{n}{2}(nk_0 - (n-1))$,

(iii) $\lambda_1 \leq (1 + k_0)n$

where k_0 is the infimum of the sectional curvatures.

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