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Construction of Noniterated Boolean Functions in the Basis $\{\&, \vee, -\}$ and Estimation of Their Number

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Abstract—In this paper we consider noniterated Boolean functions in the basis $\{\&, \vee, -\}$. We obtain the canonical form of the formula for a noniterated function in this basis. We construct the set of such formulas with respect to variables $x_1, \dots, x_n$ and calculate the number of its elements. Based on these results, we obtain the upper and lower bounds for the number of noniterated Boolean functions of n variables in the basis under consideration.

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A formula Φ over a certain basis B is said to be noniterated, if each variable enters in it no more than once. A Boolean function f is said to be noniterated in a basis B, if there exists a noniterated formula Φ over B which represents the function f.

Since the noniterated representation has the least (in all senses) complexity, the number of functions representable in a given basis B as noniterated ones is an important characteristic of this basis. Therefore, the comparison of bases with respect to the number of noniterated functions in them is not only of theoretical interest, but also of great applied importance in Microcircuit Engineering.

Some results are obtained in this direction. In [2] one tabulates the number of noniterated Boolean functions in bases $B_0 = \{\&, \vee, -\}$ and $B_1 = \{\&, \vee, \oplus, -\}$; the number of arguments of these functions does not exceed ten. This result was obtained by means of full enumeration. In papers [4] and [5] one adduces recurrent formulas for the number of noniterated Boolean functions of n variables in the same bases; these formulas allow one to calculate the mentioned number for sufficiently large n. For example, this result allows one to formulate the following conjecture: The number of noniterated Boolean functions considerably increase, when the basis B_0 is supplemented by the weakly iterated (in it) function $\oplus$. This conjecture is proved in [1] (P. 64). On the other hand, due to the recurrent nature of the obtained formulas we cannot estimate (even approximately) the number of noniterated Boolean functions of n variables for arbitrary n. The recurrent dependencies in these formulas are nonlinear; one cannot reduce them to a nonrecurrent form by the known methods.

In addition to the mentioned result, in paper [1] one describes a principally different method for constructing canonical representations of noniterated Boolean functions in the basis B_0. Based on this method, one deduces a formula for the number of noniterated Boolean functions of n variables without explicit recurrence. This formula has a complex structure; it does not allow one to obtain the desired estimates directly.

In this paper we adduce the detailed substantiation of the method for constructing the set of canonical representations for noniterated Boolean functions of variables $x_1, x_2, \dots, x_n$ in the basis B_0. This method enables one to find the desired upper and lower bounds with a similar structure. Note that an explicit formula for the number of noniterated Boolean functions of n variables is not necessary for obtaining the estimates.

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