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QUASI-BOOLEAN POWERS OF SEMILATTICES

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It is known (see [1]) that among semigroup varieties only the variety of semigroups with zero multiplication possesses the property of being closed with respect to quasi-Boolean powers. In [2], a system of axioms for a universal class of groupoids generated by quasi-Boolean powers of left-singular semigroups (semigroups of left zeros) was found. Due to reasons of duality one can obtain a characterization of the universal closure of the class of quasi-Boolean powers of right-singular semigroups (semigroups of right zeros). In the present article we describe a class of groupoids which are embeddable into quasi-Boolean powers of semilattices, which concludes the consideration of minimal non-group varieties of semigroups in this aspect.

Let L be a complete lattice. By orthogonal systems in it we understand subsets $\{l_i \mid i \in I\}$ such that $l_i \wedge l_j = 0$ for $i \neq j$ and $\bigvee\limits_{i\in I} l_i = 1$. An orthogonal system $\{l_i \mid i \in I\}$ is independent by definition if $\bigvee\limits_{j\in J} l_j \wedge \bigvee\limits_{k\in K} l_k = 0$ for any partition $I = J \cup K$, $J \cap K = \emptyset$. A complete lattice with complements is said to be quasi-Boolean if in it all the orthogonal systems are independent. In particular, all complete Boolean lattices are quasi-Boolean. A complete lattice with complements is quasi-Boolean if and only if it admits $\wedge$-homomorphism on a complete Boolean lattice which preserves the least upper bounds of orthogonal systems and with which the unique preimage of zero is zero and the unique preimage of the unit is unit (see [3], p. 271). Such a $\wedge$-homomorphism is said to be canonical.

Let $(S, \cdot)$ be a semilattice and L a quasi-Boolean lattice. The construction of L-power $S[L]$ of a semilattice S is defined as follows. It is a groupoid whose elements are all possible mappings $\nu : S \to L$ with the set of values $\{\nu(s) \mid s \in S\}$, representing an orthogonal system in L, while the multiplication is defined via the formula

$$(\mu\nu)(s) = \bigvee_{s=xy} (\mu(x) \wedge \nu(y)) \tag{1}$$

for any $\mu, \nu \in S[L]$, $s \in S$.

When L runs over the class of all quasi-Boolean lattices, we get quasi-Boolean powers of the semilattice S. Among them all the Boolean powers are situated if we restrict ourselves to complete Boolean lattices L.

The Boolean powers of a semilattice are semilattices, because Boolean powers of an algebra preserves its equational theory (see [4]). We denote by $\mathcal{U}$ the hereditary class of groupoids, which is generated by the quasi-Boolean powers of semilattices. We cite an example of a non-associative groupoid from the class $\mathcal{U}$, which is even simpler than that in [1]. Take an eight-element Boolean lattice L^* with atoms a, b, and c, dual atoms $A = a'$, $B = b'$, and $C = c'$, the least element 0 and the greatest element 1. We "split" elements a, b, A, and C by replacing them with the two-element chains $a_0 < a_1$, $b_0 < b_1$, $A_0 < A_1$, and $C_0 < C_1$, respectively. In the resulting set L we define the following coverings: The element 1 follows directly after each of the elements A_1, B,