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On the Number of Models of Theories of Locally Free Algebras

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Abstract—We study the number of prime models over finite sets and of limit models for countable complete theories of locally free algebras.

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In [1, 2] theoretical-model questions for locally free algebras was studied. The concept of the limit model was introduced in [3] in connection with the study and classification of countable models of complete theories. By the same reason the problem of existence l-Ehrenfeucht theories in different classes of theories was posed (l-Ehrenfeucht theories are theories with finitely many but greater than one limit models). In this work this problem is solved for theories of locally free algebras. We show that a theory of locally free algebra has one limit model if it is small, and has 2^ω limit models if this theory has continuum many types. We also show that a theory of locally free algebra with continuum many types has 2^ω models which are prime over finite sets.

Definition 1 ([3]). A model M is called limit if M is not a prime model over any cortege and $M = \bigcup_{n\in\omega} M_n$ for some elementary chain $(M_n)_{n\in\omega}$ of prime models over some corteges.

Definition 2. A theory T is called small if the set of its types is countable.

Definition 3 ([1, 2]). A free L-algebra is an algebra isomorphic to the algebra of all L-terms. An L-algebra A is called locally free if any finitely generated subalgebra of A is free.

Theorem 1. *The countable theory of a locally free algebra is small if and only if its signature contains no more than one single functional symbol and does not contain functional symbol of area $n \geq 2$.*

Proof. Let T be a theory of locally free unar. Since different terms correspond to the different elements, each element has no more than one preimage. Then each connected component of any model of the theory T is an infinite chain with or without an initial element. Thus, a type of any element contains the information about the number of steps which are required to obtain this type from the constants and the member having no preimages. Therefore, there are countably many types.

Suppose that the signature contains at least two symbols of single functions $f_0(x)$ and $f_1(x)$. For each word $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$ of the alphabet $\{0, 1\}$, we construct a formula $\varphi_\varepsilon(x)$ in the following way:

$$\varphi_\emptyset = x \approx x,$$
$$\varphi_\varepsilon = \exists y_1 \exists y_2 \dots \exists y_n (f_{\varepsilon_1}(y_1) \approx x \wedge f_{\varepsilon_2}(y_2) \approx y_1 \wedge \dots \wedge f_{\varepsilon_n}(y_n) \approx y_{n-1}).$$

For each $\delta \in \{0,1\}^\omega$, the set of formulas $S_\delta = \{\varphi_\varepsilon \mid \varepsilon \text{ is a finite initial section of } \delta\}$ is consistent. If $\delta \neq \delta'$, then the set $S_\delta \cup S_{\delta'}$ is inconsistent because every element has a unique preimage , i.e., it is

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