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ESTIMATION OF THE KOLMOGOROV N-WIDTHS FOR CERTAIN COMPACTS IN THE STRENGTHENED SOBOLEV SPACES

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The estimation of N-widths is necessary in optimizing the digital methods for solving elliptic problems. In the present article we consider variational problems in unusual subsets $G_{1,1}$ of the Sobolev spaces $V \equiv W_2^1(\Omega)$ ([1]–[5]). Provided with special scalar products, these subsets are the Hilbert spaces (so-called *strengthened Sobolev spaces*). In the two-dimensional case, the squared norm of $G_{1,1}$ contains integrals of squared first derivatives over segments (stringers, rods) $S_1, \dots, S_{r^*}$. More complex problems correspond to the equilibrium conditions for plates supported by rods or stringers. These problems were formulated (in the pre-Hilbert space) by S.P. Timoshenko in 1915 (see [6], [7]). The three-dimensional problems are connected, for instance, with certain hydrodynamic problems (see [8]–[10]). We cite also examples of problems on compound manifolds including blocks of different dimensions, and give estimation of the widths of corresponding compacts.

1. Strengthened Sobolev spaces $G_{1,1}$ for the two-dimensional domains

1.1. Strengthened Sobolev spaces

Let Ω be a bounded domain on the whole plane with the Lipschitz piecewise-smooth boundary $\Gamma \equiv \partial\Omega$. Let $V \equiv W_2^1(\Omega)$ be the classical Sobolev space (see [1]–[4]) with the scalar product $(u,v)_V \equiv (1, \nabla u \nabla v)_{0,\Omega} + (u,v)_{0,\Omega}$, where $(u,v)_{0,\Omega} \equiv (u,v)_{L_2(\Omega)}$, $|u|_{0,\Omega} \equiv (u,u)^{1/2}_{0,\Omega}$.

We connect a *strengthened Sobolev space* $G_{1,1,(2)} \equiv G_{1,1,(2)}(\Omega; S)$ with consideration of a subset $S \subset \overline{\Omega}$ consisting of segments (rods, stringers) $S_1, \dots, S_{r^*}$ with their end-points lying on Γ. If these segments are considered as cut lines, we obtain a partition of $\overline{\Omega}$ into blocks (panels) $P_1, \dots, P_{r'}$ with the Lipschitz piecewise-smooth boundaries ($r' = 1$ if $S \subset \Gamma$). If s stands for a local arc parameter on S_r and D_s means differentiation along S_r, $r \in [1, r^*]$, then we can define the original linear space $G \equiv G_{1,1,(2)}$ (strengthening of the space V, see [2], [3]) as a subset consisting of the functions $v \in V$ such that their traces on each S_r belong to $W_2^1(S_r)$, $r \in [1, r^*]$.

Let us provide G with the scalar product

$$(v, v')_G \equiv (v, v')_{W_2^1(\Omega)} + \sum_{r=1}^{r^*} (v, v')_{W_2^1(S_r)} \tag{1.1}$$

and with the corresponding norm $\|v\|_G$.