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DEPENDENCE OF ASYMPTOTICS OF THE MAJOR EIGENVALUES OF A TRUNCATED CONTINUUM CONVOLUTION ON THE RATE OF SYMBOL MAXIMIZATION

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In this paper, we study the asymptotic behavior of extreme eigenvalues of the operator of the truncated continuum convolution

$$[A_y f](t) = \int_0^y k(t-s)f(s)ds, \quad y > 0, \quad t \in [0, y], \tag{0.1}$$

with a real-valued symbol $K(x)$ which attains its maximum at one point or at a finite number of points.

The main results of the paper are theorems 2.1 and 2.2, where we prove that the major eigenvalues of the operator A_y tend to $M = \max\limits_{x\in\mathbb{R}} K(x)$ as $y \to +\infty$ at rate $1/y^\nu$; here $\nu > 0$ is the order of zero of the function $M - K(x)$.

This investigation is a kind of a reply to [1], where an analogous result for the operator of truncated discrete convolution is obtained only for even ν and in other cases the estimates are proved which use even numbers bordering ν. The author has relied on the basic results of papers [2]–[4] which are also described in [5] ([6] is the Russian translation of the previous edition of this book). Our method does not use them and allows one to obtain the exact by order upper-bounded and lower-bounded estimates for any $\nu > 0$.

1. Denotations

We use the following denotations: $\mathbb{N}$ is the set of natural numbers; $\mathbb{R}$, $\mathbb{C}$ are the fields of real and complex numbers, respectively; $\mathbf{i}$ is the imaginary unit, $\mathbf{e}$ is the Napierian base; μ is the Lebesgue measure on $\mathbb{R}$.

By $\mathcal{F}$ we denote the Fourier transformation which affects the function $f \in L_2(\mathbb{R})$ as follows: $[\mathcal{F}f](x) = \frac{1}{\sqrt{2\pi}} \int\limits_{\mathbb{R}} f(t)\mathbf{e}^{-\mathbf{i}xt}dt$. It is well known that the Fourier transformation is invertible and $\|f\|_{L_2} = \|\mathcal{F}f\|_{L_2}$.

For any measurable set $X \subset \mathbb{R}$ we introduce a sesquilinear form (a "scalar product") $(f, g)_X = \int\limits_X f(x)\overline{g(x)}dx$ for functions $f, g \in L_2(\mathbb{R})$, and also we do a seminorm $\|f\|_X = \sqrt{(f, f)_X}$.

Let I, I_y be the identity operators in the spaces $L_2(\mathbb{R})$ and $L_2(0, y)$, respectively.

Definition. Let $X \subset \mathbb{R}$, $f : X \to \mathbb{R}$. We say that the function f has zero of order $\nu > 0$ at a point x_0 if one can find a neighborhood U of the point x_0 and positive numbers C_1 and C_2 such that for any $x \in U - \{x_0\}$ the inequalities $C_1 \leqslant \frac{|f(x)|}{|x-x_0|^\nu} \leqslant C_2$ are fulfilled.

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