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ASYMPTOTIC THEOREMS FOR MODIFICATIONS OF POLYNOMIALS WHICH ARE SIMILAR TO THE BAERNSTEIN POLYNOMIALS

T.V. Yershova

In this article we continue the investigation of properties of modifications introduced by V.S. Videnskiĭ and T.P. Pendina to augment the order of approximation of smooth functions and applied to polynomials U_n, which are similar to the Baernstein polynomials. Asymptotics for central moments of odd order ν and orders exceeding ν of the modifications $U_{n,\nu}$ are established. The asymptotic Voronovskaya–Baernstein theorems are proved for the modifications $U_{n,\nu}$.

To prove the Khlodovskiĭ theorem on convergence of the sequence of derivatives of the Baernstein polynomials $B_n^{(k)}f$ to the derivatives $f^{(k)}$, in [1] (p. 48) polynomial operators U_n were introduced, similar to the Baernstein polynomials. The polynomials U_n, $n \in N$, are given via the formula

$$U_n(f,x) = \frac{n}{x(1-x)} \sum_{k=0}^{n} \left(\frac{k}{n} - x\right)^2 f\left(\frac{k}{n}\right) C_n^k x^k (1-x)^{n-k}, \tag{1}$$

where $f \in C[0,1]$.

Everywhere in this article x is assumed to belong to $[0,1]$. We define the modifications by Videnskiĭ and Pendina of the operators U_n via the recursion (see [2], p. 52):

$$\begin{aligned} &U_{n,1}f = U_n f, \\ &U_{n,\nu}f = U_n f - \sum_{k=1}^{\nu-1} \frac{S_k(U_n)}{k!} U_{n,\nu-k} f^{(k)} \quad \text{for} \quad \nu \ge 2, \end{aligned} \tag{2}$$

where $f \in C^{(\nu-1)}[0,1]$. We denote by $S_k(U_n)$ the functions $S_k(U_n,x) = U_n((t-x)^k, x)$, which are called central moments of order k, $k \in Z_+$, of the operators U_n. In this article we consider central moments of the Baernstein polynomials B_n, the polynomials U_n, and the modifications $U_{n,\nu}$.

From (1) it follows that

$$S_k(U_n,x) = \frac{n}{x(1-x)} S_{k+2}(B_n,x). \tag{3}$$

As is known (see [1]–[3]),

$$S_2(B_n,x) = \frac{x(1-x)}{n}, \quad S_3(B_n,x) = \frac{x(1-x)(1-2x)}{n^2}. \tag{4}$$

In view of (3) and (4) we have

$$S_0(U_n) = 1. \tag{5}$$

Applying (2) and (5), we obtain

$$S_0(U_{n,\nu}) = 1, \quad \nu \ge 1. \tag{6}$$