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Localization of the Method of Guiding Functions in the Problem about Periodic Solutions of Differential Inclusions

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Abstract—In this paper we consider the problem on the existence of forced oscillations in nonlinear objects governed by differential inclusions. We propose certain modifications of the methods of generalized and integral guiding functions.

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1. INTRODUCTION

The method of guiding functions developed by M. A. Krasnosel'skii, A. I. Perov, N. A. Bobylev, and others (see [1–3]) is one of the most efficient techniques for solving problems about periodic oscillations (see, e.g., [4–6]). In classical papers dedicated to the method of guiding functions one usually assumes that these functions are smooth on the whole phase space (see, e.g., [1–4, 7]). This condition can be limitative, for example, when the guiding potentials are different in different domains of the space.

In this paper, following the ideas of [8] and [9], we propose a certain modification of the method of guiding functions in the problem of the existence of forced oscillations in nonlinear objects described by differential inclusions. In particular, we verify the main condition for a guiding function on domains defined more "subtly " than in classical situations (cf. [1, 2, 4]).

See, e. g, [6, 7, 10] for some other generalizations of the method of guiding functions for a nonsmooth case.

Let the symbols $Kv(R^n)$ and $P(R^n)$ stand for the sets of all nonempty convex compact and nonempty subsets of R^n, respectively.

We consider a periodic problem for a differential inclusion in the following form:

$$x'(t) \in F(t, x(t)), \tag{1}$$

$$x(0) = x(T), \tag{2}$$

assuming that the multimapping $F : R \times R^n \to Kv(R^n)$ satisfies the following conditions (see, e.g., for the terminology):

(F_t) the multifunction F is T-periodic with respect to the first argument ($T > 0$):

$$F(t, x) = F(t + T, x) \quad \text{for all} \quad t \in R,\ x \in R^n$$

(evidently, this condition allows one to consider the multimapping F only on the set $[0, T] \times R^2$);

(F_1) for each $x \in R^n$ the multifunction $F(\cdot, x) : [0, T] \to Kv(R^n)$ has a measurable section;

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