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An Extremal Constrained Routing Problem With Internal Losses

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Abstract—We consider the following problem: one has to visit a finite number of sets and perform certain work on each of them. The work is accompanied by certain (internal) losses. The movements from some set to another one are constrained and accompanied by external (aggregated additively) losses. We propose a "through" variant of the dynamic programming method, formulate an equivalent reconstruction problem, and develop an optimal algorithm based on an efficient dynamic programming algorithm.

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1. INTRODUCTION

The problem under consideration goes back to the well-known traveling salesman problem (TSP) (see [1] and references therein). It includes several difficulties (in a general case the problem is NP-hard) that hamper the application of the usual exact methods (for finding the global extremum) and lead to interesting theoretical results related to the dynamic programming method (DPM). See, first of all, papers [2, 3] for the application of the DPM in the TSP. For areas related to the TSP see the following papers mentioned in [1]: 1) the TSP with a choice [4–6] or a generalized TSP (see also [7–11]); 2) an analog of the TSP with precedence conditions; in [1] it is called the courier problem (see references in [1]). In connection with item 2) see papers [12, 13], and [14–19]. In this paper we consider problems 1) and 2), taking into account that the sets (megalopolises in [20]) are connected with certain works that lead to additional (in comparison with [7–11] and [14–19]) expenses. The goal of this paper is finding the global extremum. One of the variants considered below is analogous to the dynamic travelling salesman problem in [20] (P. 45), but it is hampered by precedence conditions (see the courier problem [1]). We use the DPM for constructing efficient techniques and for studying the evolution of layers of the Bellman function in a constrained extremal problem. Certainly, the branch-and-bound method [21] (see also [20] and [22]) plays an important role in solving problems of large dimension. Constructions based on the DPM are applicable in constrained problems of small dimension. The precedence conditions are used for a more thrifty implementation of the DPM in a scheme that does not imply the construction of the whole set of values of the Bellman function [23]. Among the possible applications let us mention problems connected with sea and air transportation, where constraints can take the form of the precedence conditions. Thus, for example, in certain cases a transport facility (a ship, an airplane, or a helicopter) during the transportation process has to solve several "incidental" problems: When arriving at a certain point it should take goods or mail to be delivered to another point. In order to ensure this possibility, the arrival at the first point (that of the sender) has to precede visiting the second one (that of the receiver). Given a finite set of such sender-receiver pairs as a directive, we obtain a problem with a rather difficult restriction imposed on the choice of a route for the mentioned transport facility. Among the possible applications let us mention problems [24] that arise in nuclear power engineering; in particular, this concerns the routing problem that occurs in dismantling the worn-out power generating units.

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