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ON EXPONENTIAL STABILITY OF LINEAR DIFFERENCE-DIFFERENTIAL SYSTEMS

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1. Theorem on exponential stability

Indicators of exponential stability of linear difference-differential equations (LD-DE) contain in an explicit form the condition of stability of "neutral part", which is the invertibility of the difference operator acting on the higher derivative (see [1], items 12.4–12.6; [2], Chap. 3). The question on the necessity of this condition was studied by V.G. Kurbatov in [3], [4]. In [5] (Chap. 5), it was shown that the stability of the neutral part is a necessary condition for exponential decrease of a solution and its derivative with respect to the norm $\mathbf{L}_p$ $(1 \le p \le \infty)$. Such a property of solutions of LD-DE is called in [5] *exponential stability of LD-DE by norm* $\mathbf{W}_p^{(1)}$. Let us note that by theorem 6.3.11 of [5] (pp. 111, 112), for an LD-DE with smooth coefficients the $\mathbf{W}_p^{(1)}$-stability is equivalent to the stability by Lyapunov (i. e., stability in the Chebyshev metric).

In the present article we formulate a theorem which complements the results by V.G. Kurbatov. This theorem asserts, in particular, that for initial functions of class $\mathbf{W}_1^{(1)}$ the exponential stability by Lyapunov implies the stability of the neutral part in $\mathbf{L}_1$-metrics. Hence and from the results of [5] it follows that the exponential stability is equivalent to the $\mathbf{W}_1^{(1)}$-stability for constantly acting $\mathbf{L}_1$-perturbations. The results of this article were announced earlier in [6], [7].

In the present article we shall use the following definitions and notation. $\mathbf{R}^n$ is the Euclidean space of n-dimensional vector-columns $x = \operatorname{col}\{x_1, \dots, x_n\}$ with the scalar product $\langle x, y\rangle = \sum\limits_{i=1}^{n} x_i y_i$, the norm $\|x\| = \sqrt{\langle x, x\rangle}$, and the coordinate basis $\{e_i\}_{i=1}^n$, moreover, $E = \{e_1, \dots, e_n\}$ is the unit $n \times n$-matrix. $\mathbf{M}^n$ is the space of $n \times n$-matrices $A = \{a_{ij}\}_{i,j=1}^n$ with the operator norm $\|A\|$. $\mathbf{L}_1(\Delta)$ is the space (of classes of equivalence) of measurable functions $z : \Delta \to \mathbf{R}^n$ which are summable by the Lebesgue measure on the interval $\Delta \subset \mathbf{R}$, provided with the norm $\|z\|_{\mathbf{L}_1(\Delta)} = \int\limits_{\Delta} \|z(t)\| dt$; $\mathbf{L}_\infty(\Delta)$ is the space (of classes of equivalence) of vector-functions $y : \Delta \to \mathbf{R}^n$ which are measurable and bounded in essential on the interval $\Delta \subset \mathbf{R}$, provided with the norm $\|z\|_{\mathbf{L}_\infty(\Delta)} = \operatorname{vrai\,sup}\limits_{t\in\Delta} \|z(t)\|$; $\mathbf{W}_1(\Delta) = \mathbf{W}_1^{(1)}(\Delta)$ is the space of functions $x : \Delta \to \mathbf{R}^n$ which are absolutely continuous on the interval $\Delta \subset \mathbf{R}$ and bounded with respect to the norm $\|x\|_{\mathbf{W}_1(\Delta)} = \|x\|_{\mathbf{L}_1(\Delta)} + \|\dot x\|_{\mathbf{L}_1(\Delta)}$.

We denote by $\mathbf{B}^*$ the space conjugate to the linear normed space $\mathbf{B}$, by $Q^* : \mathbf{B}^* \to \mathbf{B}^*$ — the operator adjoint to a linear bounded operator $Q : \mathbf{B} \to \mathbf{B}$. The space $(\mathbf{L}_1(\Delta))^*$ is identified with the space $\mathbf{L}_\infty(\Delta)$ by means of the canonical duality

$$(z, y) = \int_\Delta \langle z(t), y(t)\rangle dt, \quad z \in \mathbf{L}_1(\Delta), \quad y \in \mathbf{L}_\infty(\Delta).$$

Everywhere in what follows χ_Δ is the characteristical function of the set Δ.

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