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GENERATION OF TWO- AND THREE-DIMENSIONAL GRIDS FOR PROBLEMS OF GAS DYNAMICS ON THE BASE OF THE POISSON EQUATION

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Two approaches are known to generate the computational grids in two-dimensional and three-dimensional domains with fixed boundary — the variational and finite-difference ones. The second of them uses a mapping of the computation domain with physical variables x, y, z onto a rectangular domain in the space of curvilinear coordinates ξ, η, ζ. The vector function $\vec{r}(\vec{\xi}) = (x(\vec{\xi}), y(\vec{\xi}), z(\vec{\xi}))$ ($\vec{\xi} = (\xi, \eta, \zeta)$) which is the radius-vector of a point in physical space, is sought as a solution of elliptic vector equations. The simplest patterns are the Laplace and Poisson equations of second order. The right sides of the latter one (so-called "control functions") serve for the control over the distribution of grid nodes in computation domain.

1. Generation of two-dimensional grids

In order to describe the generation of two-dimensional grids we choose the vector Poisson equation of the form [1]

$$g_{22}(\vec{r}_{\xi\xi} + P\vec{r}_{\xi}) + g_{11}(\vec{r}_{\eta\eta} + Q\vec{r}_{\eta}) - 2g_{12}\vec{r}_{\xi\eta} = 0, \tag{1}$$

where $\vec{r}_{\xi} = \partial\vec{r}/\partial\xi$, $\vec{r}_{\xi\xi} = \partial^2\vec{r}/\partial\xi^2$, $g_{ij} = \vec{a}_i \cdot \vec{a}_j = \vec{r}_{\xi} \cdot \vec{r}_{\eta}$ are coefficients of metric tensor. The control functions P, Q for equation (1) serve for concentration (or rarefaction) of the coordinate lines of the family $\xi = \text{const}$ (in the case of P) and $\eta = \text{const}$ (in the case of Q) around chosen lines and points. The function P has the following form:

$$P = -\sum_{i=1}^{N} a_i \operatorname{sign}(\xi - \xi_i) e^{-c_i|\xi - \xi_i|} - \sum_{k=1}^{M} b_k \operatorname{sign}(\xi - \xi_k) e^{-d_k((\xi-\xi_k)^2 + (\eta-\eta_k)^2)^{1/2}},$$

$Q(\xi, \eta) = P(\eta, \xi)$. The first terms serve for either contraction ($a_i > 0$) or rarefaction ($a_i < 0$) of the coordinate lines $\xi = \text{const}$ near chosen lines $\xi = \xi_i$, $i = 1, 2, N$. The second terms correspond to either contraction ($b_k > 0$) or rarefaction ($b_k < 0$) of the coordinate lines $\xi = \text{const}$ to the chosen grid nodes (ξ_k, η_k), $k = 1, 2, M$, on these coordinate lines. The value of constants a_i, b_k determines the intensity of contraction. The constants $c_i, d_k > 0$ define the size of contraction–rarefaction domain: the smaller a coefficient is, the greater is the number of coordinate lines $\xi = \text{const}$ being under the influence of this term of the control function.

We approximate the vector equation in partial derivatives (1) by finite difference equations (for the first and second derivatives we use symmetric differences). Then we solve the obtained system of linear equations by means of the simple iteration method. According to the form of computation domain, we construct initial approximation by either transfinite interpolation, or interpolation along one of the coordinate directions. We use as the boundary value conditions either the distribution of nodes on the boundary immediately (i. e., the Dirichlet boundary value