A chiral cosmological model in modified thery of gravitation

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Historical development (physical basis)

- 1957. **Julian Schwinger**, *A Theory of Fundamental Interactionas.* Annals of Physics: **2**, 407-437 (1957).
  "σ - field", "σ - particle"
- 1958. **T.H.R. Skyrme**, *A non-linear theory of strong interactions.* Proc.Roy.Soc., A, **247**, 260-278 (1958).
  "σ - field" denoted as "ϕ_4", $\phi_i, (i = 1, 2, 3)$ correspond to π^+, π^0, π^- mesons
- 1960. **M. Gell-Mann & M. Levy**, *Axial vector current in beta decay.* Nuovo Cim., **26**, 705 (1960).
  Terms "linear" and "non-linear" sigma models

Historical development (mathematical basis)

Definition Nonlinear sigma model or Harmonic map $\iff$ Multiple scalar field with kinetic interaction

The action of a nonlinear sigma model (NSM):

$$S = \int \sqrt{-g} d^4 x \left(\frac{1}{2} h_{AB} \partial_\mu \varphi^A \partial_\nu \varphi^B g^{\mu\nu} \right) \tag{1}$$

- $g_{\mu\nu}(x)$ – a metric of a space-time $\mathcal{M}$,
- h_{AB} – a metric of a target or chiral space $\mathcal{N}$,
- $\varphi = (\varphi^1, \dots, \varphi^N)$ – a multiplett of the chiral fields,
- $\varphi^A_{,\mu} = \partial_\mu \varphi^A = \frac{\partial \varphi^A}{\partial x^\mu}$

Historical development (mathematical basis)

- 1954 **F.B. Fuller**, *Proc.Natl.Acad.Sci.* **40** 987 (1954). Definition and term.
- 1964 **J. Eells, J.H. Sampson,** *Harmonic mappings of riemannian manifolds.* American J.Math., v.**86**, No.1, p.109 (1964). Development of the theory. Obtained solutions include instanton one, obtained later in 1975 by Belavin and Polyakov.
- 1968 **J. Eells, L. Lemaire**, *Bull. London Math. Soc.* **10** 1 (1968). Development of the harmonic maps theory.
- 1978 **C.W. Misner**, *Phys.Rev.* **D 18** 4510 (1978). Possible applications of harmonic maps in physical theories.

Historical development (physical basis)

- 1975. **A.A. Belavin, A.M. Polyakov**, *Instanton solutions in the SO(3) 2D NSM.* JETP Lett., **22**, No. 10, 503 (1975)
- 1975 **A.M.Polyakov**, *Phys.Lett.* **B59** (1975) 79 (1975).
- 1976 **A.A.Migdal**, *Soviet Phys.Jetp* **42** (1976) 413, 742.
- 1976. **K. Pohlmeyer**, *Integrable hamiltonian systems and interactions constraints.* Comm.Math.Phys., v.**46**, p.207 (1976). Soliton solutions in the SO(3) 2D NSM
- 1978. **D.J. Gross**, *Meron configurations in the two-dimentional O(3) sigma-model.* Nucl.Phys., v.**B132**, p.439 (1978). Meron solutions in the O(3) 2D NSM

Selfgravitating NSM

- 1979. **V. De'Alfaro, S. Fubini, G. Furlan**, Nuovo Cim., **A50**, 523 (1979). Instanton+meron solutions in the SO(3) 4D NSM, coupled to gravity. Signature of $\mathcal{M}$ is $(++++)$
- 1983. **G.G. Ivanov**, Theor. & Math.Phys. **50**, 45 (1983) Chiral fields as the source of the gravitational field. Signature of $\mathcal{M}$ is $(+---)$. SO(3) solutions in 4D self-gravitating NSM
- 1983. **S.V. Chervon**, *SO(4) solutions in 4D self-gravitating NSM*, Russian Phys.J., New York, **26**, No.8, 89 (1983).
- 1986. **S.V. Chervon**, *The exact solutions of self-gravitating SO(N)-invariant NSM*. Grav. & GR (Kazan: Kazan State University), **23**, 103 (1986). SO(N) solutions in 4D self-gravitating NSM + method of exact solution generating.

Effective NSM

- 1967. **R.A. Matzner, C.W. Misner**, *Gravitational field equations for sources with axial symmetry and angular momentum.* Phys.Rev. **154**, 1229 (1967). Vacuum Einstein eqs. as dynamic eqs. of NSM.
- 1989 **S.V. Chervon, A.G. Muslimov**, *Plane-symmetric gravitational field as a four-component nonlinear sigma model. Phys.Lett.* **A 142** 14 (1989)
- 1992 **S.V. Chervon**, *On the chiral model for non-linear scalar fields.* Grav. & GR **29**, 85 (1992). Self-interacting scalar field as 2-component NSM
- 1998 **A. Ashtekar, V. Husain**, *Int.J.Mod.Phys*, **D7**, 549 (1998)
- 2000 **S.V. Chervon, D.Yu. Shabalkin**, *A Plane-Symmetric Gravitational Field with Matter as a Generalized NSM. Grav. & Cosmol.* **6** No 1 41 (2000)

Chiral cosmological model

- 1993 **S.V. Chervon**, *The massive chiral model on inflation arena.* Int. Conference on Astrophysics and Cosmology, December 20-23, 1993, Saha Institute of Nuclear Physics, Calcutta, India. Abstracts, p.44 (1993).
- 1995. **S.V.Chervon**, *On the chiral inflationary model.* Russ. Phys. J., New York, **38**, 539 (1995)
- 1995. **S.V.Chervon**, *Chiral non-linear sigma models and cosmological inflation.* Grav. & Cosm. **1**, No.2, 91 (1995)
- 1997. **S.V. Chervon**, *Non-linear fields in theory of gravitation and cosmology.* Ulyanovsk State University, Ulyanovsk, 1997 -191p. (ISBN 5-7769-0037-9) Cosmological perturbations for chiral inflationary model.

The action of CCM

The action of a chiral cosmological model:

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{2\kappa} + \frac{1}{2} h_{AB} \partial_\mu \varphi^A \partial_\nu \varphi^B g^{\mu\nu} - V(\varphi) \right) \quad (2)$$

- R – a scalar curvature, $V(\varphi)$ – a potential,
- κ – Einstein's gravitational constant,
- $g_{\mu\nu}(x)$ – a metric of a space-time $\mathcal{M}$,
- h_{AB} – a metric of a target or chiral space $\mathcal{N}$,
- $\varphi = (\varphi^1, \ldots, \varphi^N)$ – a multiplett of the chiral fields,
- $\varphi^A_{,\mu} = \partial_\mu \varphi^A = \frac{\partial \varphi^A}{\partial x^\mu}$

Energy-momentum tensor

The Energy-momentum tensor of the model (2):

$$T_{\mu\nu} = \varphi_{A,\mu}\varphi^A_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}\varphi^A_{,\alpha}\varphi^B_{,\beta}g^{\alpha\beta}h_{AB} - V(\varphi)\right). \tag{3}$$

The Einstein equation can be reduced to the form

$$R_{\mu\nu} = \kappa(h_{AB}\varphi^A_{,\mu}\varphi^B_{,\nu} - g_{\mu\nu}V(\varphi)) \tag{4}$$

The dynamic equations of the chiral fields are:

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\varphi^{,\mu}_A) - \frac{1}{2}\frac{\partial h_{BC}}{\partial\varphi^A}\varphi^C_{,\mu}\varphi^B_{,\nu}g^{\mu\nu} + V_{,A} = 0, \tag{5}$$

where $V_{,A} = \frac{\partial V}{\partial \varphi^A}$.

Space-time

The metric of homogeneous and isotropic Universe in the Friedman – Robertson – Walker (FRW) form reads:

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \epsilon r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right). \qquad (6)$$

- $a(t)$ – a scalar factor,
- $\epsilon = \{0, -1, +1\}$ – a curvature characteristic,
- $x^i = (r, \theta, \varphi)$ – the spatial spherical coordinates
- $x^0 = t$ – cosmic time, $c = 1$.

The 2-component model

The target space metric is

$$ds_\sigma^2 = h_{11}(\phi,\psi)d\phi^2 + 2h_{12}(\phi,\psi)d\phi d\psi + h_{22}(\phi,\psi)d\psi^2. \qquad (7)$$

- $\varphi^1 = \phi,\ \varphi^2 = \psi$ – a notation of chiral fields,
- $h_{11}(\phi,\psi),\ \ h_{12}(\phi,\psi),\ \ h_{22}(\phi,\psi)$ – the chiral metric components
- $h_{12} = 0,\ \ h_{11} > 0$ then ϕ is a canonical scalar field,
- $h_{12} = 0,\ \ h_{11} < 0$ then ϕ is a phantom scalar field,

The 2-component model

Einstein equations read:

$$H^2 = \frac{\kappa}{3}\left[\frac{1}{2}h_{11}\dot{\phi}^2 + h_{12}\dot{\phi}\dot{\psi} + \frac{1}{2}h_{22}\dot{\psi}^2 + V(\phi,\psi)\right] - \frac{\epsilon}{a^2}, \quad (8)$$

$$\dot{H} = -\kappa\left[\frac{1}{2}h_{11}\dot{\phi}^2 + h_{12}\dot{\phi}\dot{\psi} + \frac{1}{2}h_{22}\dot{\psi}^2\right] + \frac{\epsilon}{a^2}, \quad (9)$$

- $\dot{a} := \frac{da}{dt}$
- $H = (\dot{a}/a)$ – Hubble parameter,
- $a(t) > 0, \quad \dot{a} > 0$ – expansion of the Universe
- $V = V(\phi, \chi)$ – a potential of (self)interactions

The 2-component model

The chiral fields equations are:

$$3H\left(h_{11}\dot{\phi}+h_{12}\dot{\psi}\right)+\partial_t\left(h_{11}\dot{\phi}+h_{12}\dot{\psi}\right)$$
$$-\frac{1}{2}\frac{\partial h_{11}}{\partial\phi}\dot{\phi}^2-\frac{\partial h_{12}}{\partial\phi}\dot{\phi}\dot{\psi}-\frac{1}{2}\frac{\partial h_{22}}{\partial\phi}\dot{\psi}^2+\frac{\partial V}{\partial\phi}=0, \qquad (10)$$
$$3H\left(h_{12}\dot{\phi}+h_{22}\dot{\psi}\right)+\partial_t\left(h_{12}\dot{\phi}+h_{22}\dot{\psi}\right)$$
$$-\frac{1}{2}\frac{\partial h_{11}}{\partial\psi}\dot{\phi}^2-\frac{\partial h_{12}}{\partial\psi}\dot{\phi}\dot{\psi}-\frac{1}{2}\frac{\partial h_{22}}{\partial\psi}\dot{\psi}^2+\frac{\partial V}{\partial\psi}=0. \qquad (11)$$

- $h_{11}=\pm 1$ – the choice of gaussian coordinates,
- $h_{12}=0$ – no cross interaction between fields,
- $V=V(\phi,\chi)$ – a potential of (self)interactions

The 2-component model

Useful implications of Einstein equations:

$$\frac{1}{2}h_{11}\dot{\phi}^2(t) + h_{12}\dot{\phi}(t)\dot{\psi}(t) + \frac{1}{2}h_{22}(t)\dot{\psi}^2(t) = \frac{1}{\kappa}\left[\frac{\epsilon}{a^2} - \dot{H}\right], \quad (12)$$

$$V(t) = \frac{3}{\kappa}\left(H^2 + \frac{1}{3}\dot{H} + \frac{2}{3}\frac{\epsilon}{a^2}\right). \quad (13)$$

The consequence for analysis of Universe acceleration parameter

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{3}\left[\frac{1}{2}h_{11}\dot{\varphi}^2(t) + h_{12}\dot{\varphi}(t)\dot{\psi}(t) + \frac{1}{2}h_{22}(t)\dot{\psi}^2(t) - V\right]. \quad (14)$$

Phantonical Model

The parametrization of the quintom model is

$$h_{11} = -1, \quad h_{22} = 1, \quad h_{12} = 0,$$

Generalization includes the kinetic interaction term

$$h_{11} = -1, \quad h_{22} = h_{22}(\phi), \quad h_{12} = 0,$$

where h_{22} is a given function on the first chiral field ϕ.

- **T. Matos and L.A. Urena-Lopez, 2000**. Quintessence and scalar dark matter.
- **G-B. Zhao, J-Q. Xia, M. Li, B. Feng and X. Zhang, 2005**. Quintom model.
- **C. Van de Bruck and J.M. Weller, 2009**. Two scalar fields and mixed kinetic term.

SO(3) phantonical model

Parametrization of phantonical analog of SO(3) nonlinear sigma model

$$h_{11} = -1, \quad h_{22} = \sin^2\phi, \quad h_{12} = 0. \tag{15}$$

The exact solution for $\epsilon = 0, \quad V = const = V_* = \Lambda$

The well-known solution is

$$H = \sqrt{\frac{\Lambda}{3}} \tanh(\sqrt{3\Lambda} t), \tag{16}$$

$$a = a_* [\cosh(\sqrt{3\Lambda} t)]^{1/3}. \tag{17}$$

SO(3) phantonical model

The solution for the first (phantom) field can be obtained from the expression

$$\cos\phi = -\frac{\sqrt{C_1^2 + 2\Lambda}}{\sqrt{2\Lambda}} \sin\left(\sqrt{\frac{2}{3}} \arctan\left(\sinh(\sqrt{3\Lambda} t)\right) + C_2\right) \tag{18}$$

The second (canonical) field is

$$\psi - \psi_0 = \frac{1}{2a_*}\left[\ln\left|\frac{C_1}{\sqrt{2\Lambda}}\tan z + 1\right| - \ln\left|\frac{C_1}{\sqrt{2\Lambda}}\tan z - 1\right|\right] \tag{19}$$

$$z = \sqrt{\frac{2}{3}}\arctan(\sinh(\sqrt{3\Lambda}t)) + C_2 \tag{20}$$

Inflaton – DS Fields Model

The action of the model is:

$$S = \int \sqrt{-g} d^4 x \left[\frac{R}{2\kappa} + \frac{1}{2} \phi_{,\mu} \phi_{,\nu} g^{\mu\nu} - V(\phi) + \right. \\ \left. + \frac{1}{2} h_{AB}(\varphi^C) \varphi^A_{,\mu} \varphi^B_{,\nu} g^{\mu\nu} - W(\varphi^C) \right] \quad (21)$$

- ϕ – an inflationary field (inflaton),
- $V(\phi)$ – a self-interaction potential for inflaton,
- h_{AB} – a metric of a target or chiral space $\mathcal{N}$,
- $\varphi = (\varphi^1, \ldots, \varphi^N)$ – a multiplett of the chiral fields,
- $W(\varphi)$ – a (self)interaction potential for DS fields

Approximations

The total potential of the model:

$$W_{tot}(\phi, \varphi^C) = V(\phi) + W(\varphi^C). \quad (22)$$

The total kinetic energy in FRW metric:

$$\mathrm{K}_{tot}(\phi, \varphi^C) = K_\phi + K_\sigma = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}h_{AB}\dot{\varphi}^A\dot{\varphi}^B. \quad (23)$$

Approximations:

- $W(\varphi^C) \ll V(\phi), \quad W_{tot}(\phi, \varphi^C) \approx V(\phi),$
- $K_\sigma \ll K_\phi, \quad \mathrm{K}_{tot}(\phi, \varphi^C) \approx K_\phi$

Approximated Equations

Spacetime metric in conformal time:

$$ds^2 = a^2(\eta)(-d\eta^2 + \gamma_{ij}dx^i dx^j), \tag{24}$$

Equations for self-gravitating inflaton are:

$$\mathcal{H}' - \mathcal{H}^2 = -\kappa K_\phi, \tag{25}$$

$$3\mathcal{H}^2 = \kappa K_\phi + a^2 V(\phi), \tag{26}$$

$$\phi'' + 2\mathcal{H}\phi' + a^2 V_{,\phi} = 0. \tag{27}$$

- $\mathcal{H} = \frac{a'}{a} = \frac{1}{a}\frac{da}{d\eta}$
- γ_{ij} – the metric of 3D section.

The 2-component NSM

The chiral metric is:

$$ds_\sigma^2 = d\psi^2 + h_{22}(\psi,\chi)d\chi^2 \tag{28}$$

Equations for the chiral fields are:

$$\psi'' + 2\mathcal{H}\psi' \mp \psi\chi'^2 + a^2 W_{,\psi} = 0, \tag{29}$$

$$\psi^2(\chi'' + 2\mathcal{H}\chi') + 2\psi\psi'\chi' + a^2 W_{,\chi} = 0. \tag{30}$$

- $h_{22} = \ \pm\psi^2$, " $-$ " sign related to phantom field,
- $W_{,\varphi^A} = \dfrac{\partial W}{\partial \varphi^A}$.

Einstein's perturbed equations are:

$$\delta G^{\nu}_{\mu} = \kappa(\delta T^{\nu}_{\mu} + \Theta^{\nu}_{\mu}), \quad \delta T^{\nu}_{\mu} \simeq \Theta^{\nu}_{\mu} \tag{31}$$

where

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} - V(\phi)\right), \tag{32}$$

$$\Theta_{\mu\nu} = \psi_{,\mu}\psi_{,\nu} + h_{22}\chi_{,\mu}\chi_{,\nu} - - g_{\mu\nu}\left(\frac{1}{2}\psi_{,\alpha}\psi^{,\alpha} + \frac{1}{2}h_{22}\chi_{,\alpha}\chi^{,\alpha} - W(\psi,\chi)\right), \tag{33}$$

$$\Theta^{0}_{0} = \rho = a^{-2}K_{\sigma} + W, \quad \Theta^{*}_{*} = -p = -a^{-2}K_{\sigma} + W. \tag{34}$$

Longitudinal gauge

The space time metric in longitudinal gauge:

$$ds^2 = a^2(\eta)\left\{(1+2\Phi)d\eta^2 - (1-2\Phi)\gamma_{ij}dx^i dx^j\right\}. \qquad (35)$$

Perturbed equations in longitudinal gauge:

$$\begin{aligned}
&\nabla^2\Phi - 3\mathcal{H}\Phi' - (\mathcal{H}' + 2\mathcal{H}^2)\Phi = \frac{\kappa}{2}(\phi_0'\delta\phi' + a^2 V_{,\phi}\delta\phi + \Theta_0^0),\\
&\Phi' + \mathcal{H}\Phi = \frac{\kappa}{2}\phi_0'\delta\phi, \qquad (36)\\
&\Phi'' + 3\mathcal{H}\Phi' + (\mathcal{H}' + 2\mathcal{H}^2)\Phi = \frac{\kappa}{2}(\phi_0'\delta\phi' - a^2 V_{,\phi}\delta\phi - \Theta_*^*),
\end{aligned}$$

- $\Phi = \Phi(\eta, \vec{r})$, ϕ_0 – solution of background eqs.
- $\delta\phi = \delta\phi(\eta, \vec{r})$

Perturbations

The consequence for gravitational perturbation $\Phi = \Phi(\eta, \vec{r})$:

$$\Phi'' - \nabla^2\Phi + 2\Phi' \left(\mathcal{H} - \frac{\phi_0''}{\phi_0'} \right) + 2\Phi \left(\mathcal{H}' - \mathcal{H}\frac{\phi_0''}{\phi_0'} \right) + \kappa W = 0. \quad (37)$$

Perturbation of the inflaton:

$$\delta\phi = \frac{2}{\kappa\phi_0'}(\Phi' + \mathcal{H}\Phi). \quad (38)$$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2 V_{,\phi\phi}\delta\phi = 0. \quad (39)$$

- $\phi_0, \mathcal{H}$ – obtained from background eqs. (25)-(27),
- $\Phi = \Phi(\eta, \vec{r})$ – obtained from (37),
- $\delta\phi = \delta\phi(\eta, \vec{r})$ – obtained from (38).

Inflation

Inflationary stage:

$$a(t) = a_s e^{h_* t} (a_s, h_* - constants) \qquad (40)$$

In conformal time presentation:

$$a(\eta) = -\frac{1}{h_* \eta}, \quad \eta = -\frac{e^{-h_* t}}{a_s h_*} \qquad (41)$$

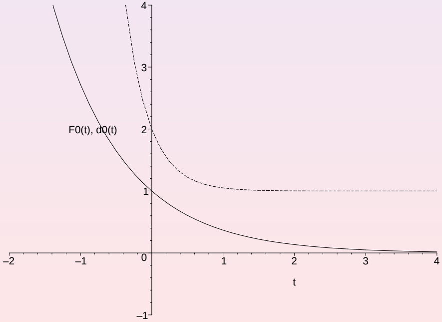
The solution of (25)-(27) is:

$$\mathcal{H}(\eta) = -\frac{1}{\eta}, \quad \phi_0 = const, \quad V(\phi) = const. \qquad (42)$$

- $\eta : \eta \in (-\infty, -1/[a_s h_*])$ when $t : t \in (0, \infty)$,
- $\Phi(\eta, \mathbf{x}) = \Phi(\eta) e^{i\mathbf{kx}}$.
- $|\vec{k}| = k \ll 1$ – long-wavelength approximation

Diagram 0

Φ (solid) and $\delta\phi$ (dashed) lines vs. cosmic time t in standard inflation



The solution 1

In the case of inflation eq. (37) reads:

$$\Phi'' - \nabla^2\Phi + \kappa W(\eta) = 0. \tag{43}$$

A Higg's type potential is:

$$W(\psi) = \pm\frac{h_*^2\gamma^2}{4\beta^2}\psi^4 + h_*^2\psi^2 + W_* \tag{44}$$

The solutions of (29)-(30) are:

$$\psi = \beta\eta, \tag{45}$$

$$\chi = \gamma\eta + const. \tag{46}$$

- W_*, β, γ – constants,

The solution 1

In terms of conformal time:

$$W(\eta) = h_*^2\beta^2\eta^2(1 \pm \frac{\gamma^2}{4}\eta^2) + W_*, \qquad (47)$$

$$K(\eta) = \frac{1}{2}h_*^2\beta^2\eta^2(1 \pm \gamma^2\eta^2). \qquad (48)$$

The solution with accounting of DS fields:

$$\Phi_1 = C_2\eta \mp \frac{\kappa(h_*\beta\gamma)^2}{120}\eta^6 - \frac{\kappa(h_*\beta)^2}{12}\eta^4 - \frac{\kappa W_*}{2}\eta^2. \qquad (49)$$

The solution 1

Equation of state:

$$w = \frac{K_\sigma - W}{K_\sigma + W} = \frac{p_\sigma}{\rho_\sigma}. \tag{50}$$

- $w < -1/3$ – DE condition,
- $t = 2$ – time of the inflation end.

Diagram 1

The potential W, kinetic energy K, EOS w vs. cosmic time t

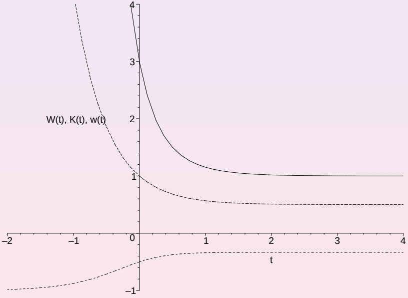


Diagram 1ph

The potential W, kinetic energy K, EOS w vs. cosmic time t for phantom field χ

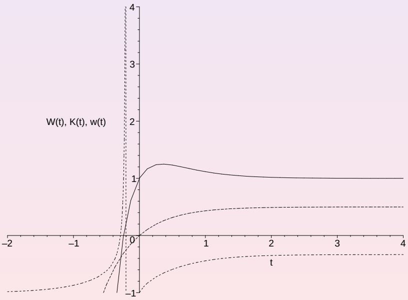


Diagram 0-3

Perturbations of the gravitational field Φ vs. cosmic time t for 0,1,2,3 solutions

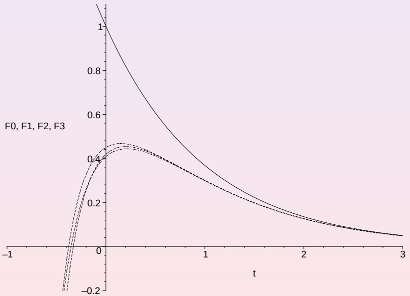
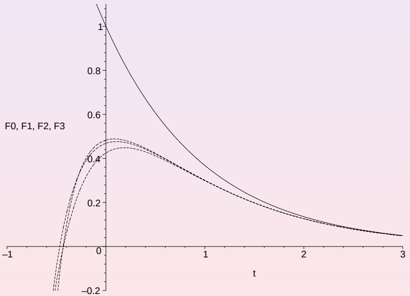


Diagram 0-3ph

Perturbations of the gravitational field Φ vs. cosmic time t for 0,1,2,3 solutions for phantom χ



IDS model on PL background

Power law inflation:

$$a = a_* t^m, \quad m > 1$$

for the cosmic time t.

$$a = a_s \eta^\alpha, \quad \alpha = \frac{m}{1-m}, \quad \alpha < -1$$

for the conformal time η .
The relations between cosmic and conformal times:

$$\eta = \frac{t^{1-m}}{a_s(1-m)}, \; t = [a_s(1-m)\eta]^{\frac{1}{1-m}}$$

then $t \to 0, \eta \to -\infty$ and $t \to \infty, \eta \to 0$.

IDS model on PL background

The background solution is

$$\mathcal{H}(\eta) = \frac{\alpha}{\eta}, \quad \phi = \sqrt{\frac{2}{\kappa}\alpha(\alpha+1)}\ln\eta + \phi_0, \tag{51}$$

$$V(\phi) = \frac{\alpha(2\alpha-1)}{a_s\kappa}\exp\left(-\frac{(\alpha+2)(\phi-\phi_0)}{\sqrt{\frac{2}{\kappa}\alpha(\alpha+1)}}\right). \tag{52}$$

The gravitational perturbation Φ can be obtained from eq.

$$\Phi'' - \nabla^2\Phi + 2\Phi'\left(\frac{\alpha+1}{\eta}\right) + \kappa W = 0. \tag{53}$$

IDS model on PL background

- W as the function on dark sector fields ψ and χ ?
- h_{22} as the function on dark sector fields ψ and χ ?

Let us set

$$h_{22} = \psi^2, \quad \psi = \beta\eta$$

$$W = -K_1^2\psi^{\lambda_1} + K_2^2\psi^{\lambda_2} + W_0.$$

Where K_1, K_2, λ_1 and λ_2 are constants which should be matched with the model parameters.
Thus the solutions for the dark sector fields are:

$$\psi = \beta\eta, \quad \chi = \chi_0 - \frac{1}{(2\alpha+1)}\frac{1}{\eta^{(2\alpha+1)}}. \qquad (54)$$

IDS model on PL background

Constants for the solution are

$$K_1^2 = \frac{\beta^{2(\alpha+1)}}{a_s^2}, \;\; \lambda_1 = -2\alpha, \;\; K_2^2 = \frac{2C_1^2\beta^{6\alpha}}{a_s^2(1-2\alpha)}, \;\; \lambda_2 = 1-2\alpha \quad (55)$$

Neglecting by the term $\nabla^2\Phi$ in the long-wavelength approximation we can solve the equation (53) exactly. The gravitational perturbation is

$$\Phi = \frac{\kappa\beta^2}{6a_s^2(1-\alpha)}\eta^{2(1-\alpha)} - \frac{2\kappa C_1^2\beta^{4\alpha+1}}{4a_s^2(1-2\alpha)(3-2\alpha)}\eta^{(3-2\alpha)} - C_2\frac{\eta^{-(2\alpha+1)}}{(2\alpha+1)}. \quad (56)$$

IDS model on PL background

For example taking $m = 3$ one can obtain the potential of dark sector fields

$$W = -\frac{1}{a_s^2\beta}\psi^3 + \frac{C_1^2}{2a_s^2\beta^9}\psi^4 + W_0. \tag{57}$$

The gravitational perturbation (53) reads

$$\Phi = \frac{\kappa\beta^2}{15a_s^2}\eta^5 - \frac{\kappa C_1^2}{48a_s^2\beta^5}\eta^6 + \frac{C_2}{2\eta^2}. \tag{58}$$

Thus we can see the influence of the expansion parameter α and dark sector field parameter β for the gravitational perturbation and the potential W.

IDS model on PL background

The comparison with standard solution for long-wavelength approximation (the formula (5.65) in Mukhanov et al, 1992)

$$\Phi^{(0)} = A\eta^{1+\alpha}, \; A = const \tag{59}$$

stress the more crucial difference then in the case with exponential inflation. Namely there are no terms in (53) corresponding to standard solution (59) for any admitted α. Thus taking into account the dark sector fields influence for gravitational perturbation we may obtain a very different picture for structure formation.

CCM coupled to Perfect Fluid

The action

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}h_{AB}\partial_\mu\varphi^A\partial_\nu\varphi^B - V(\varphi^C)\right) + S_{(pf)} \quad (60)$$

where $S_{(pf)}$ is the action of perfect fluid. Einstein equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_P^2}(T^{(\sigma)}_{\mu\nu} + T^{(pf)}_{\mu\nu}) \quad (61)$$

where $c = 1, \kappa = 8\pi G = M_P^{-2}$

Energy-momentum tensor (EMT) of perfect fluid is

$$T^{(pf)}_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \quad (62)$$

where p – pressure, ρ – energy density, u^μ – 4-velocity of elementary volume of liquid.

2-CCM coupled to Perfect Fluid

The chiral space metric is

$$ds_\sigma^2 = h_{11}(\varphi, \chi)d\varphi^2 + h_{22}(\varphi, \chi)d\chi^2 \tag{63}$$

(Note! $\phi \to \varphi, \quad \psi \to \chi$)

The conservation law for perfect fluid

$$\nabla^\mu T_{\mu\nu}^{(pf)} = 0 \tag{64}$$

takes in FRW Universe the form

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{65}$$

$CCM - \sigma$CDM models

Cosmological models with cold dark matter

- ΛCDM model
- Quintessence QCDM model: $h_{11} = 1, \quad h_{22} = 0$
- Phantom CDM model: $h_{11} = -1, \quad h_{22} = 0$
- Quintom qCDM model: $h_{11} = -1, \quad h_{22} = 1$
- New! σQCDM model: $h_{11} = 1, \quad h_{22} = \exp\{f(\varphi, \chi)\}$
- New! σqCDM model: $h_{11} = 1, \quad h_{22} = -\exp\{f(\varphi, \chi)\}$

The model equations

If dust-like matter consist from few components with the density ρ_j the Friedmann equation is

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{1}{3M_P^2} \sum_j \rho_j \tag{66}$$

Income to the matter density in the Universe

$$\Omega = \frac{\rho}{3M_P^2 H^2} = \frac{\rho}{\rho_{crit}} \tag{67}$$

Deceleration parameter is

$$q = -\frac{\ddot{a}a}{(\dot{a})^2} \tag{68}$$

The model equations

To be in agreement with the model by **S. Unnikrishnan, H. K. Jassal, T. R. Seshadri, 2008**, we choose

$$h_{11} = 1, \; h_{22} = \pm \exp\left(2\frac{(\varphi - \varphi_i)}{M_P}\right) \tag{69}$$

$$V = \exp\left(-\sqrt{\lambda}\frac{\varphi}{M_P}\right) + \exp\left(-\sqrt{\lambda}\frac{\chi}{M_P}\right) \tag{70}$$

Friedmann eq. with account of CDM density ρ_m has the view:

$$H^2 = \frac{1}{3M_P^2}\left[\rho_m + \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2 + V(\varphi, \chi)\right] \tag{71}$$

The model equations

Chiral fields eqs. are

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{2}\frac{\partial h_{22}}{\partial \varphi}\dot{\chi}^2 + \frac{\partial V}{\partial \varphi} = 0 \quad (72)$$

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{h_{22}}\frac{\partial h_{22}}{\partial \varphi}\dot{\varphi}\dot{\chi} + \frac{1}{h_{22}}\frac{\partial V}{\partial \chi} = 0 \quad (73)$$

The contributions from CDM and DS chiral fields are

$$\Omega_m = \frac{\rho_m}{3M_P^2 H^2} = \frac{\rho_m}{\rho_{crit}}, \quad (74)$$

$$\Omega_{DS} = \frac{1}{3M_P^2 H^2}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2 + V\right) = \frac{\rho_{DS}}{\rho_{crit}} \quad (75)$$

The perturbations

The definition of the perturbation

$$\varphi(t) = \varphi(t) + \delta\varphi(t, \bar{x}),$$

$$\chi(t) = \chi(t) + \delta\chi(t, \bar{x})$$

The longitudinal gauge

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)\delta_{ij}dx^i dx^j \tag{76}$$

General view of perturbed eqs. for chiral fields and for gravitation field in the longitudinal gauge can be found in (Chervon, 2002, 2013).

The density contrast

$$\delta_{DS} = \frac{-\delta T^0_0}{-T^0_0} = \left[\frac{1}{2}\frac{\partial h_{22}}{\partial \varphi}\delta\varphi\dot{\chi}\dot{\chi} + \delta\dot{\varphi}\dot{\varphi} + h_{22}\delta\dot{\chi}\dot{\chi} - \Phi\dot{\varphi}^2 - \Phi h_{22}\dot{\chi}^2 + \frac{\partial V}{\partial \varphi}\delta\varphi + \frac{\partial V}{\partial \chi}\delta\chi \right] \left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2 + V \right)^{-1} \quad (77)$$

$$\delta_m = \frac{\delta\rho_m}{\rho_m} = \frac{1}{\rho_m}\left(\frac{1}{8\pi G} 2\left(-3H^2\Phi - 3H\dot{\Phi} + \nabla^2\Phi \right) - \left[\frac{1}{2}\frac{\partial h_{22}}{\partial \varphi}\delta\varphi\dot{\chi}\dot{\chi} + \delta\dot{\varphi}\dot{\varphi} + h_{22}\delta\dot{\chi}\dot{\chi} - \Phi\dot{\varphi}^2 - \Phi h_{22}\dot{\chi}^2 + \frac{\partial V}{\partial \varphi}\delta\varphi + \frac{\partial V}{\partial \chi}\delta\chi \right] \right) \quad (78)$$

σQCDM and σqCDM

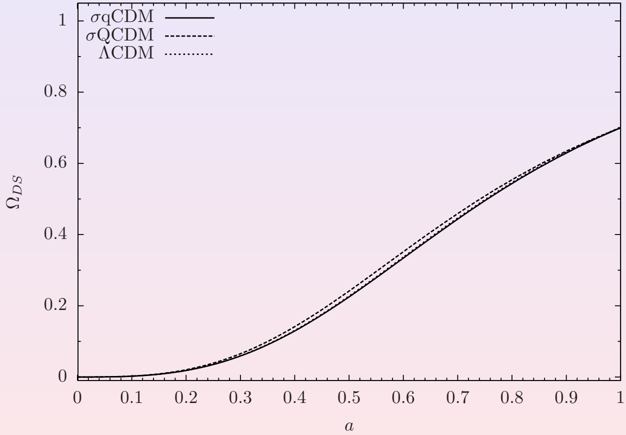


Figure : DE Contributions

σQCDM and σqCDM

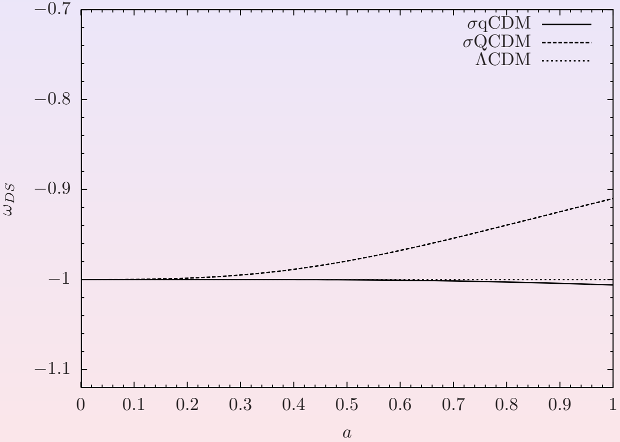


Figure : DE EoS

σQCDM and σqCDM

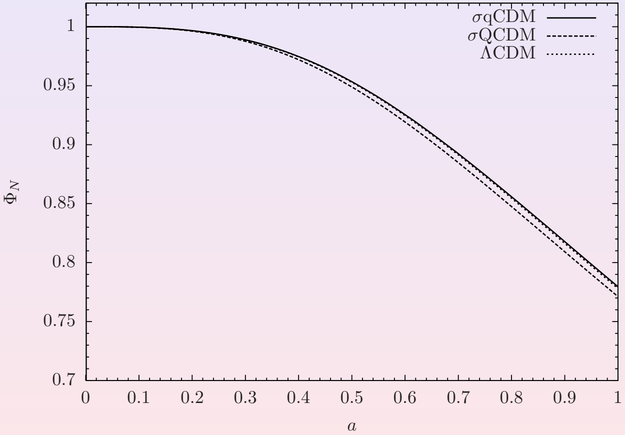


Figure : Metric perturbation

σQCDM and σqCDM

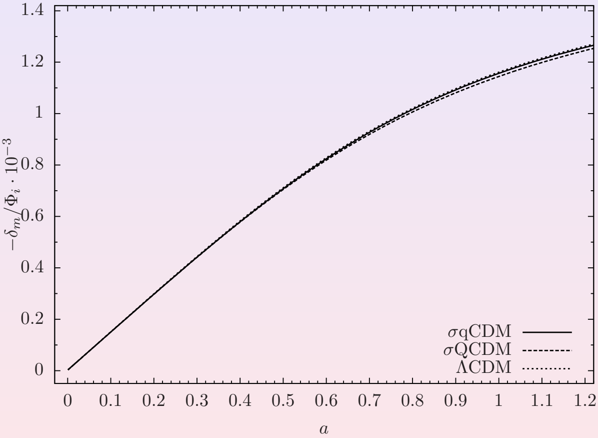


Figure : Dark matter density perturbation

σQCDM and σqCDM

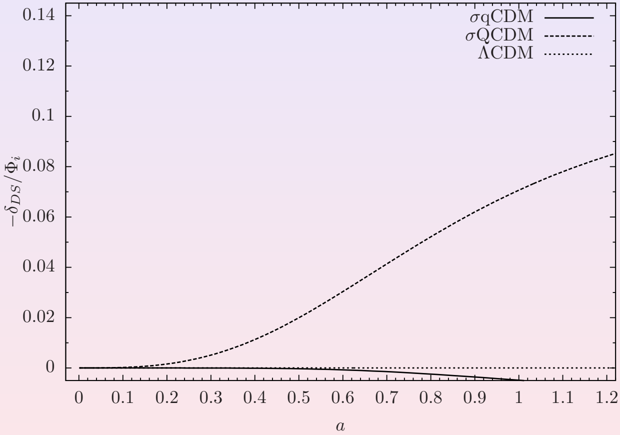


Figure : Dark energy density perturbation

σQCDM and σqCDM

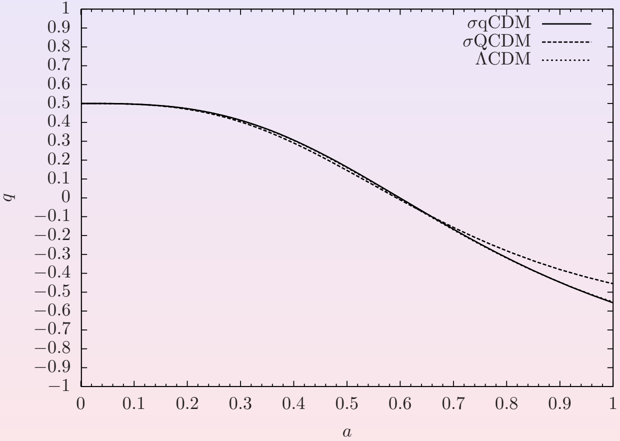


Figure : The deceleration parameter

Summary of σCDM models

- The models constructed are similarly to ΛCDM in most parameters.
- The very different behaviour is for σQCDM model in equation of state parameter and for DE perturbation.
- The kinetic interaction between DS fields has an effect on DE and DM parameters and their perturbations.
- The study of more general chiral cosmological model can lead to better agreement with observational data.

The reconstruction of the chiral metric and the potential

Let us investigate CCM with the potential

$$V = V(\varphi).$$

The solution of the second field eq. is

$$\dot{\chi}^2 = \frac{C_1^2}{h_{22}^2 a^6} \quad \text{or, in equivalent form} \quad h_{22}\dot{\chi}^2 = \frac{C_1^2}{h_{22} a^6}$$

If we set $h_{22} \sim a^{-3}$, then $h_{22}\dot{\chi}^2 \sim a^{-3}$ — this behavior is adequate to DM.

The reconstruction of the chiral metric and the potential

For the sake of generality let us set dependence of the kinetic terms of CCM by ansatz

$$\frac{1}{2}h_{22}\dot{\chi}^2 = g(a)$$

$$\frac{1}{2}\dot{\varphi}^2 = f(a)$$

The second field may represent dust-like matter when

$$h_{22} = a^{-3}.$$

For the first field we choose the simple presentation

$$g(a) = \frac{1}{2}\dot{\varphi}^2 = B$$

The reconstruction of the chiral metric and the potential

Now we are looking for expression of Hubble parameter which will be suitable for confrontation with observation data. By renormalization of parameters we obtain

$$\tilde{H}^2 = \frac{H^2}{H_0^2} = \tilde{\Lambda} - 6\tilde{B}\ln a + 2\tilde{C}a^{-3} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4},$$

where

$$\tilde{B} = \frac{B}{\rho_c}, \quad \tilde{C} = \frac{C}{\rho_c}, \quad \tilde{\Lambda} = \frac{\Lambda}{\rho_c}, \quad \tilde{H}^2 = \frac{H^2}{H_0^2}.$$

$$\tilde{\Lambda} = 1 - 2\tilde{C} - \Omega_{b0} - \Omega_{r0} = \Omega_{\sigma\Lambda}, \quad \Omega_{\sigma cdm0} = 2\tilde{C}, \quad \Omega_{m0} = \Omega_{\sigma cdm0} + \Omega_{b0}.$$

Finally we have

$$\tilde{H}^2 = \Omega_{\sigma\Lambda} - 6\tilde{B}\ln a + \Omega_{\sigma cdm0}a^{-3} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4}.$$

The reconstruction of the chiral metric and the potential

The difference from ΛCDM model is in the presence of the term $6\tilde{B}\ln a$

$$\Omega_\Lambda = 1 - \Omega_b - \Omega_r - \Omega_{cdm}$$

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \Omega_{cdm} a^{-3} + \Omega_b a^{-3} + \Omega_r a^{-4}$$

Reconstruction will be performed using the expression

$$\int_0^a H_0 \varphi' da = \int_0^a \frac{f(a)da}{\sqrt{2}a\tilde{H}}, \quad a = a(\varphi)$$

Observational data used

WMAP Collaboration (E. Komatsu et al.),
Astrophys.J.Suppl. 192 (2011) 18
Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation.

SDSS Collaboration (Will J. Percival et al.).,
Mon.Not.Roy.Astron.Soc. 401 (2010) 2148-2168
Baryon Acoustic Oscillations in the Sloan Digital Sky Survey Data Release 7 Galaxy Sample.

N. Suzuki et al.,
Astrophys.J. 746 (2012) 85
The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above z¿1 and Building an Early-Type-Hosted

χ^2 — goodness (badness) of fit

Supernova Ia type

$$m - M = 5\log_{10}\left(\frac{d_L}{Mpc}\right) + 25$$

d_L —luminosity distance

m —apparent magnitude

M —absolute magnitude

$$d_L(z) = \frac{1+z}{H_0}\int_0^z \frac{d\tilde{z}}{E(\tilde{z})}, \quad E(z) \equiv H(z)/H_0.$$

$$H^2(z) = H_0^2\left[\Omega_m^{(0)}(1+z)^3 + \Omega_{DE}^{(0)}\exp\left\{\int_0^z \frac{3(1+\omega_{DE})}{1+\tilde{z}}d\tilde{z}\right\}\right]$$

χ^2 — goodness (badness) of fit

Supernovae Union2.1 $0.01 < z_i < 1.5$

$$\mu(z_i) = 5\log_{10}[D_L(z_i)] + \mu_0, \quad D_L = H_0 d_L, \quad \tilde{H} = H/H_0$$

$$\mu_0 = 5\log_{10}\left[\frac{H_0^{-1}}{Mpc}\right] + 25 = 42.38 - 5\log_{10} h, \quad H_0 = \frac{h}{2998} Mpc^{-1}.$$

$$\chi^2_{SN} = \sum_{i=1}^{N} \frac{[\mu_{obs}(z_i) - \mu(z_i)]}{\sigma_i^2(z_i)}.$$

N. Suzuki et al.,

Astrophys.J. 746 (2012) 85

The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above z¿1 and Building an Early-Type-Hosted Supernova Sample

http://supernova.lbl.gov/Union/

χ^2 — goodness (badness) of fit

SDSS Collaboration (Will J. Percival et al.),
Mon.Not.Roy.Astron.Soc. 401 (2010) 2148-2168
Baryon Acoustic Oscillations in the Sloan Digital Sky Survey Data Release 7 Galaxy Sample.

Inclusion of the data on baryon acoustic oscillation and CMB

$$D_A = (1+z)^{-2} d_L(z), \text{ angular diameter distance}$$

$$D_V \equiv \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3}, \text{ effective distance measure}$$

$$\chi^2_{BAO} = \left(\frac{D_V(z=0.35)/D_V(z=0.2) - 1.736}{0.065} \right)^2 .$$

CMB data

WMAP Collaboration (E. Komatsu et al.)

$$l_A \equiv (1+z_*)\frac{\pi D_A(z_*)}{r_s(z_*)}, \text{ acoustic scale}$$

$$r_s(z) = \frac{1}{\sqrt{3}}\int_0^{1/(1+z)} \frac{da}{a^2 H(a)\sqrt{1+(3\Omega_b/4\Omega_\gamma)a}}, \text{ comoving sound horizon}$$

$$z_* = 1048[1+0.00124(\Omega_b h^2)^{-0.738}][1+g_1(\Omega_m h^2)^{g_2}]$$

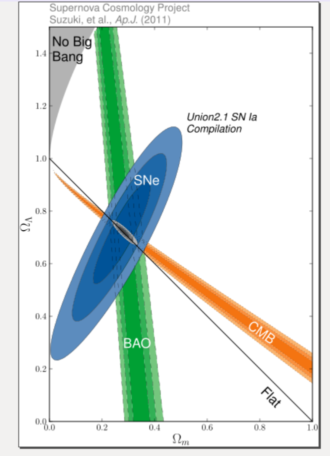
decoupling epoch

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1+39.5(\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1+21.1(\Omega_b h^2)^{1.81}}$$

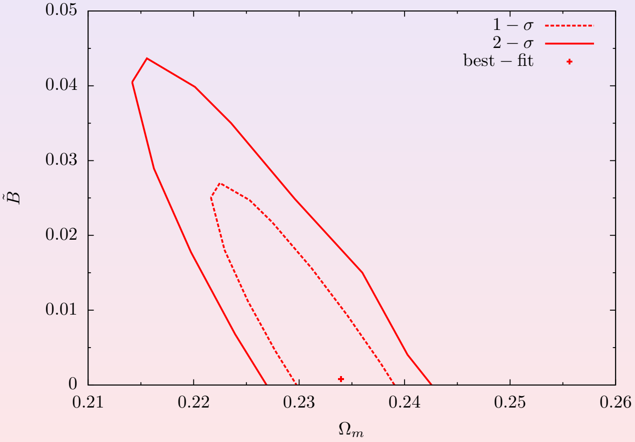
$$\chi^2_{CMB} = (x_i^{th} - x_i^{obs})(C^{-1})_{ij}(x_j^{th} - x_j^{obs})$$

$$x_i = (l_A, R, z_*), \ (C^{-1})_{ij} \text{ — WMAP7 covariance matrix}$$

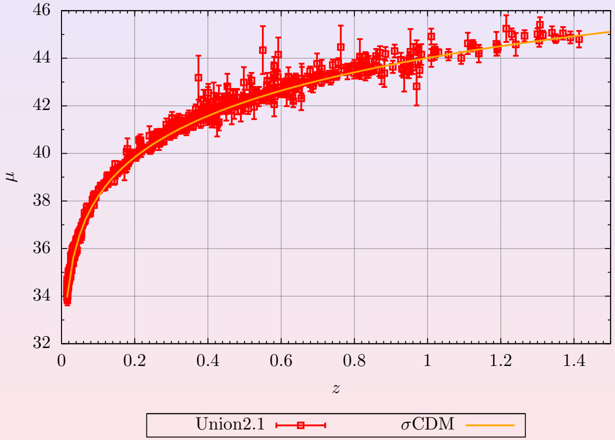
DE and DM relation at present epoch



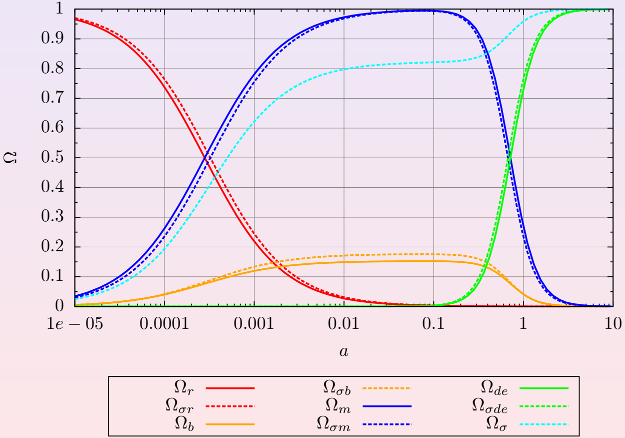
Contour plots corresponding to 1σ (68%) and 2σ (95%) likelihood levels for σCDM parameters



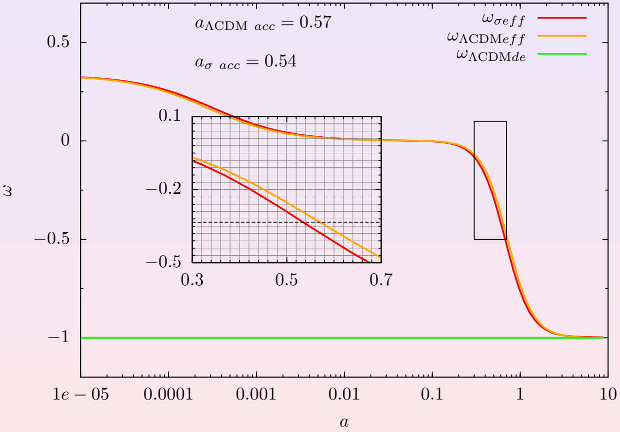
Confrontation to supernova data



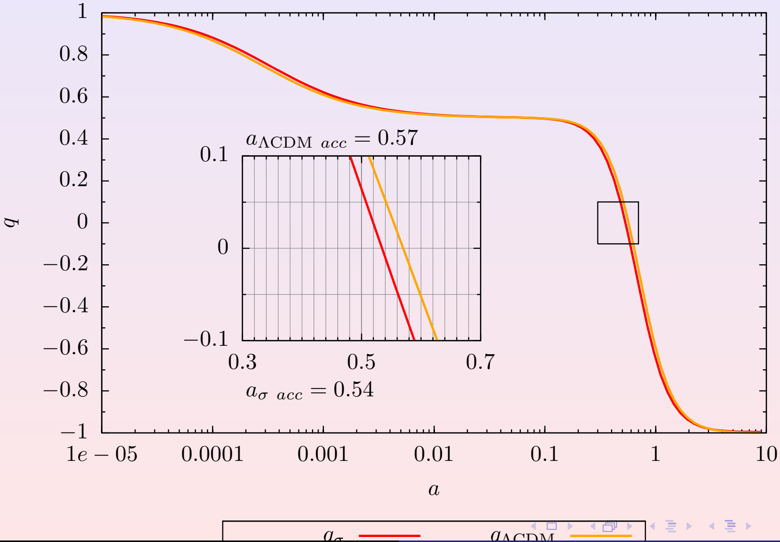
Evolution of contributions to critical density of various components in ΛCDM and σCDM models



Evolution of the effective EoS parameter



Deceleration parameter



The reconstruction of the chiral metric and the potential

The potential can be reconstructed from the relation

$$V = \Lambda - 6B \ln a + Ca^{-3} - B,$$

Chiral metric component may be defined from $h_{22} = a^{-3}$.

Now we consider two possibilities. First is approximation for Early Universe ($a \ll 1$)

$$H_0 \varphi(a) = \int_0^a \frac{da}{\sqrt{2} a \sqrt{\frac{\Omega_r}{a^4}}} = \frac{B}{\sqrt{2\Omega_r}} \frac{a^2}{2}$$

The reconstruction of the chiral metric and the potential

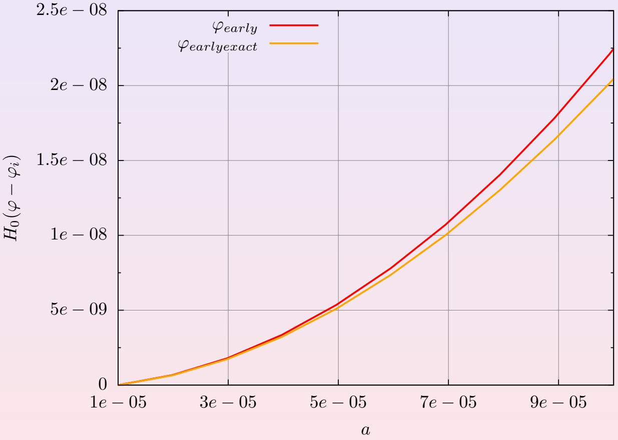
Inverting the dependence $\varphi = \varphi(a)$, we find the dependence of chiral metric from scalar field

$$h_{22} = \left(\frac{H_0\sqrt{2\Omega_{r0}}}{\sqrt{\tilde{B}}} \left(\varphi - \tilde{\varphi}_{early}\right) \right)^{-3/2}$$

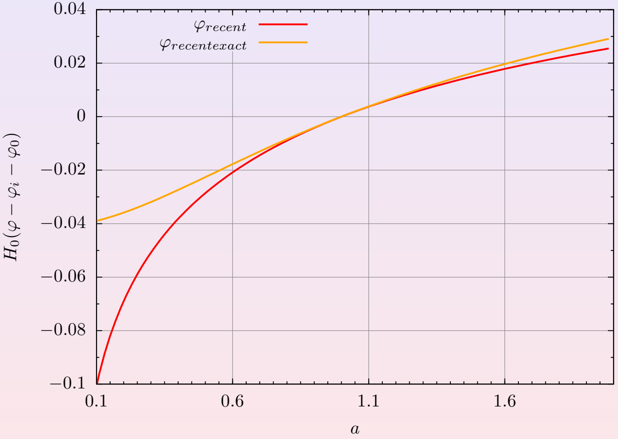
and the same for the potential

$$V = V_0 - 3B \ln \left[\frac{H_0\sqrt{2\Omega_{r0}}}{\sqrt{\tilde{B}}} \left(\varphi - \tilde{\varphi}_{early}\right) \right] + C \left[\frac{H_0\sqrt{2\Omega_{r0}}}{\sqrt{\tilde{B}}} \left(\varphi - \tilde{\varphi}_{early}\right) \right]^{-3/2} .$$

Early Universe approximation



Late Universe approximation



Perspectives

- Variation of σCDM parameters $(\lambda,\ \mu)$ shows deviation of EoS parameter from -1 and nonegligible DE perturbations for some values of parameters.
- Inclusion of radiation in σCDM gives opportunity to consider more early Universe.
- Investigation of the Universe evolution with reconstructed chiral metric and the potential.