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Solvability of Thermoviscoelastic Problem for Leray Alpha-Model

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Abstract—We investigate solvability problem (in a weak sense) for one initial boundary-value problem describing alpha-Leray model with viscosity depending on a temperature.

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Viscous incompressible fluid motion at moderate speeds with constant density is subject to the following system of equations called a *Navier–Stokes system* (NSS):

$$\begin{gathered}\frac{\partial}{\partial t}v - \nu\Delta v + (v\cdot\nabla)v + \nabla p = f,\\ \nabla\cdot v = 0,\end{gathered} \tag{1}$$

here v is the fluid particle velocity at the point x at the time moment t, p is the medium pressure, f is the exterior forces density, $\nu > 0$ is the medium viscosity. The first proof of NSS weak solvability in the bounded domain $\Omega \subset \mathbb{R}^n$, $n = 2, 3$ with a sufficiently smooth boundary $\partial\Omega$ is due to J. Leray [1]. This proof is based on system (1) regularization. Here we consider one particular case of this regularization, namely the *alpha-Leray model*:

$$\begin{gathered}\frac{\partial}{\partial t}v - \nu\Delta v + (u\cdot\nabla)v + \nabla p = f,\\ \nabla\cdot v = 0,\\ v = u - \alpha^2\Delta u,\end{gathered} \tag{2}$$

here α is a fixed positive parameter called a *model "scale subgrid (filter) length"* (both the model motivation and the necessary references can be found in [2]).

Here we consider the alpha-Leray model (2) with temperature dependent viscosity. As the temperature increases, system (2) acquires an additional energy balance equation [3]. So in this paper we study the following initial-boundary value problem:

$$\begin{gathered}\frac{\partial}{\partial t}v - 2\mathrm{Div}(\nu(\theta)\mathcal{E}(v)) + (u\cdot\nabla)v + \nabla p = f,\\ \nabla\cdot v = 0,\\ v = u - \alpha^2\Delta u,\\ \frac{\partial}{\partial t}\theta + (u\cdot\nabla)\theta - \chi\Delta\,\theta = 2\nu(\theta)\mathcal{E}(v) : \mathcal{E}(v) + g;\\ v|_{t=0} = v_0, \quad \theta|_{t=0} = \theta_0, \quad v|_{[0,T]\times\partial\Omega} = 0, \quad \theta|_{[0,T]\times\partial\Omega} = 0.\end{gathered} \tag{3}$$

Here $\theta(t,x)$ is the medium temperature, $\chi > 0$ is the thermal conductivity coefficient, g is the external heat source, $\mathcal{E}(v) = (\mathcal{E}_{ij})$, $\mathcal{E}_{ij} = \frac{1}{2}(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$, is the deformation velocities tensor, v_0 and θ_0

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