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Description of Finite Nilpotent Groups of the 2nd Degree with a Prime Odd Period

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Abstract—We describe the mentioned groups, assigning values to centralizers of the generating elements and defining a group operation on corteges of elements of the Galois field with a prime odd characteristic. On the indicated groups we define odules over the Galois field and describe these odules. We prove that the considered groups are those with unique extraction of root.

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Finite p-groups are nilpotent. Below we consider only groups, whose period p is a prime odd number and whose nilpotency degree equals 2. Groups with a 2-generated centralizer are described in [1, 2]. Below we adduce an algorithm which describes the mentioned groups with an arbitrary centralizer, enumerating the values of commutators of generating elements of the groups. In addition, these groups are defined by operations on tuples of scalars from the Galois field with a prime odd characteristic. We consider odules over the Galois field on the studied groups, thus we also describe these odules. The latter are used for the construction of finite planes [3]. The use of odules enables one to prove that finite nilpotent groups of degree 2 with m generating elements are groups with unique rooting.

We write a group operation in the additive form. We denote a group by the symbol G, we do its centralizer by G', we do its center by $Z(G)$; we write elements of a group in lowercase Greek letters; the symbol ϑ stands for the zero element of a group. The group centralizer belongs to its Frattini subgroup, therefore elements of the centralizer are nongenerating elements of the group. We denote by $\langle\alpha,\beta,\dots,\eta\rangle$ the subgroup generated by elements $\alpha,\beta,\dots,\eta$; $\langle\gamma\rangle$ is a cyclic subgroup.

Cyclic subgroups have the following trivial properties: If $\upsilon\in\langle\omega\rangle$ and $\upsilon\neq\omega,\upsilon\neq\vartheta$, then $\langle\upsilon\rangle=\langle\omega\rangle$; if $\upsilon\notin\langle\omega\rangle$, then $\langle\upsilon\rangle\cap\langle\omega\rangle=\langle\vartheta\rangle$.

We use the fact that a nilpotent group has a nontrivial center; in a group of degree 2 the group centralizer is located at its center.

Commutators of elements of the 2nd degree nilpotent groups have the following properties: $[\sigma+\tau,\rho]=[\sigma,\rho]+[\tau,\rho]$; $[t\sigma,s\rho]=ts[\sigma,\rho]$, t, s are integer numbers.

1. GROUPS WITH A CYCLIC CENTRALIZER

Let $\mathbf{G}^m$ stand for a group generated by m elements; let $\mathbf{G}'^m$ denote its centralizer. If $\mathbf{G}^2=\langle\alpha,\beta\rangle$, then $[\beta,\alpha]\neq\vartheta$. Let $[\beta,\alpha]=\tau$ and let an element τ be nongenerating for $\mathbf{G}^2$. One can write each element ρ of the group $\mathbf{G}^2$ in the form $\rho=\alpha^x\beta^y\tau^z$, where numbers x,y,z range from 0 to $p-1$. The order of the group $\mathbf{G}^2$ equals p^3, $\mathbf{G}'^2=Z(\mathbf{G}^2)=\langle\tau\rangle$. The group $\mathbf{G}^2$ is a unique 2-generated one.

Lemma 1. *The group $\mathbf{G}^3$ contains the set of generating elements $\{\alpha,\beta,\gamma\}$ such that $[\alpha,\beta]\neq\vartheta$, $[\gamma,\alpha]=[\gamma,\beta]=\vartheta$.*

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