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THE THEOREM ON EQUICONVERGENCE FOR DIFFERENTIAL OPERATORS OF HIGHER ORDER WITH A SINGULARITY

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Introduction

Let is consider the differential operator

$$ly \equiv y^{(n)} + \sum_{j=0}^{n-2} \Big(\frac{\nu_j}{x^{n-j}} + q_j(x) \Big) y^{(j)}, \quad 0 < x < T, \quad n = 2m. \tag{1}$$

Let $\mu_1, \dots, \mu_n$ be the roots of the characteristic polynomial

$$\delta(\mu) = \sum_{j=0}^{n} \nu_j \prod_{k=0}^{j-1} (\mu - k), \quad \nu_n = 1, \quad \nu_{n-1} = 0.$$

For the sake of definiteness we assume $\mu_k - \mu_j \neq sn$ $(s = 0, \pm 1, \pm 2, \dots)$; $\operatorname{Re} \mu_1 < \dots < \operatorname{Re} \mu_n$. Write $\vartheta_{nj} := 0$, $j = \overline{0, n-2}$, if $\nu_k = 0$, $k = \overline{0, n-2}$, otherwise $\vartheta_{nj} := n - 1 - \operatorname{Re}(\mu_n - \mu_1) - j$, and suppose that $q_j(x) x^{\vartheta_{nj}} \in \mathcal{L}(0, T)$. Under fulfillment of these conditions we shall say that $l \in V$.

Consider a non-selfconjugated boundary value problem $\mathcal{L}$ of the following form:

$$ly = \lambda y, \quad 0 < x < T, \quad l \in V, \tag{2}$$

$$y(x) = O(x^{\mu_{m+1}}), \quad x \to 0, \tag{3}$$

$$V_j(y) \equiv y^{(\tau_j)}(T) + \sum_{k=0}^{\tau_j - 1} v_{jk} y^{(k)}(T) = 0, \quad j = \overline{1, m}, \quad 0 \leq \tau_j \leq n-1, \quad \tau_j \neq \tau_s \ \ (j \neq s). \tag{4}$$

In the present article we shall obtain a theorem on equiconvergence of the expansion into the Fourier series with respect to eigen- and adjoint functions for the boundary value problems of the form (2)–(4) inside a finite interval $(0, T)$.

The differential equation (2) was studied in [1], [2], where special fundamental systems of solutions were constructed, asymptotics of the Stokes factors was obtained, and the inverse problem was investigated. In [3], the boundary value problem $\mathcal{L}$ was considered, the asymptotic behavior of eigenvalues was investigated, the properties of the Green function of the boundary value problem were studied, a theorem on the completeness of the system of eigen- and adjoint functions was proved, a theorem on expansion and a theorem on equiconvergence on the interval $[0, T]$ were obtained.

The presence of a singularity in the differential operator conditions brings essential difficulties in the proof of the main theorem of the present article. The method which we shall apply in

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