ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2014, Vol. 58, No. 8, pp. 71–73. © Allerton Press, Inc., 2014.
Original Russian Text © M.A. Aukhadiev, A.S. Nikitin, A.S. Sitdikov, 2014, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2014, No. 8, pp. 86–89.

A Crossed Product of The Canonical Anticommutation Relations Algebra in The Cuntz Algebra

M. A. Aukhadiev*, **A. S. Nikitin****, and **A. S. Sitdikov*****
(Submitted by A.M. Bikchentaev)

Kazan State Power Engineering University, ul. Krasnosel'skaya 51, Kazan, 420066 Russia
Received March 24, 2014

Abstract—We show that the Cuntz algebra can be represented as a crossed product of the canonical anticommutation relations algebra by an endomorphism.

DOI: 10.3103/S1066369X1408009X

Keywords: *Cuntz algebra, crossed product, recursive fermion system, C^*-algebra, isometry.*

There is a necessity to construct a field algebra [1–3] for describing the superselection structure of finite quantum fermion systems [4]. This brief communication is motivated by that problem. To this end, a crossed product of the Cuntz algebra by an endomorphism is constructed in [5] where the results from [6] are used. It should be noted that in [6] embeddings of the canonical anticommutation relations (CAR) algebra of fermions into the Cuntz algebra $\mathcal{O}_2$ are presented by means of recursive constructions.

Recall that the Cuntz algebra $\mathcal{O}_d$ ($d \geq 2$) is a C^*-algebra generated by isometries $\psi_1, \psi_2, \dots, \psi_d$ satisfying the following relations:

$$\psi_i^* \psi_j = \delta_{i,j} I,$$
$$\sum_{i=1}^{d} \psi_i \psi_i^* = I,$$

where I denotes the unit of this algebra. For the sake of convenience, we make use of the following notation: $\psi_{i_1 i_2 \cdots i_m} \equiv \psi_{i_1}\psi_{i_2}\cdots\psi_{i_m}$, $\psi^*_{i_1 i_2\cdots i_m} \equiv \psi^*_{i_m}\dots\psi^*_{i_2}\psi^*_{i_1}$ and $\psi_{i_1\cdots i_m;\, j_n\cdots j_1} \equiv \psi_{i_1}\cdots\psi_{i_m}\psi^*_{j_n}\cdots\psi^*_{j_1}$. It follows from the above-mentioned relations that the algebra $\mathcal{O}_d$ is generated as a linear space by so-called *monomials*, i.e., by the operators of the form $\psi_{i_1\cdots i_m;\, j_n\cdots j_1}$.

The *canonical* unital $*$-endomorphism ρ of the algebra $\mathcal{O}_d$ is defined by

$$\rho(X) = \sum_{i=1}^{d} \psi_i X \psi_i^*, \quad X \in \mathcal{O}_d.$$

Being a C^*-algebra, the CAR algebra of fermions possesses generators a_m and a_n^* ($m, n = 1, 2, \dots$) satisfying the conditions (see [7])

$$\{a_m,\, a_n\} = \{a_m^*,\, a_n^*\} = 0,$$
$$\{a_m,\, a_n^*\} = \delta_{m,n} I.$$

It is shown in [6] that the CAR algebra is isomorphic to the algebra $\mathcal{O}_2^{U(1)} \subset \mathcal{O}_2$ consisting of those elements of the algebra $\mathcal{O}_2$ which are invariant under the standard action of the group $U(1)$. In other words, this subalgebra is generated as a linear space by the monomials of the form

$$\psi_{i_1}\cdots\psi_{i_k}\psi^*_{j_k}\cdots\psi^*_{j_1},$$

*E-mail: m.aukhadiev@gmail.com.
**E-mail: drnikitin@rambler.ru.
***E-mail: airat_vm@rambler.ru.