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ERROR ESTIMATES FOR APPROXIMATE SOLUTIONS OF PROBLEMS OF THE LINEAR THERMOELASTICITY THEORY

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1. Introduction

Many real problems which arise in applications are described by complex systems. The latter may include differential, integral, and algebraic equations. The accuracy control problem for approximate solutions of such systems is rather difficult, because each variable of the system is measured inaccurately, which yields the further errors of the rest variables of the problem. In this paper, we consider one typical problem of this class. It arises in the joint analysis of thermal and force fields which appear when elastic bodies are heated. The temperature stresses which govern additional deformations and strains are defined using a solution of the heat conductivity problem adjoined to equations of the elasticity theory.

The control of the accuracy of approximate solutions is one of the most important problems of computing mathematics. This problem can be solved with the help of both a priori and a posteriori approaches. A priori estimates usually require additional regularity of a precise solution and demonstrate an asymptotic behavior, i. e., they indicate the rate at which the error decreases when the specific mesh parameter does. A posteriori estimates were proposed later then a priori ones. They appear due to the necessity to indicate the error distribution for an approximate solution constructed at a concrete mesh. A posteriori error indicators are widely used in modern computer programs which implement the principle of consequent mesh adaptation. A complete solution of a posteriori control problem requires not only to obtain the error distribution indicator, but also to construct a guaranteed upper bound of an approximation error in a natural (energetic) norm.

A posteriori error estimates for approximate solutions obtained by the finite-element method were studied by many authors. The so-called "residual" method was proposed first (see, e. g., [1]–[3]). Later it became prevailing. From the mathematical point of view, this method implies the calculation of the norm of the residual of the corresponding differential equation in the distribution space. This method is applicable only to Galerkin approximations. Later for the indication of the error distribution for finite-element solutions the gradient averaging method was proposed (see, e. g., [4], [5]). This method is also applicable only to Galerkin approximations and requires the higher regularity of the precise solution of the problem.

Majorants of the energetic norm of the deviation from the precise solution applicable to any functions from the energetic class are described in [6]–[8]. These majorants were obtained by the general methods of functional analysis and duality theory for the calculus of variations, therefore, they can be called a posteriori estimates of the functional type. They were investigated for problems of the linear elasticity theory in [9], [8].

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