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A STATISTICAL ESTIMATION FOR THE MAXIMAL EIGENVALUE OF MATRIX

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The problem of optimal calculation of the maximal eigenvalue for a matrix of higher dimension has numerous applications in Engineering Mathematics, Graph Theory and other areas (see, e. g., [1]–[5]).

In the present we article we suggest a new procedure for approximate calculation of the maximal eigenvalue for a matrix with non-negative elements. The obtained results answer in part for problem 5 of the book [6] (see § 9.5).

We describe theoretical base of the procedure under consideration in terms of Markov chain theory and estimate the error of approximation. Then we cite an example of the numerical realization of this procedure for a concrete matrix, to which an application of the direct methods of computation of its eigenvalues proves to be non-effective.

Section 1 contains theoretical results necessary to prove the proposed procedure. In Section 2 we shall describe both the procedure by itself and an example of its realization. Section 3 contains the proof of results of Section 1.

1. A statistical estimation of the maximal eigenvalue of fragment of jump probabilities

Let $\{X_i,\ i \geq 1\}$ be a Markov's chain with a finite set of states S and jump probabilities $\|p_{ij}\|$, A be a certain subset of S. We understand moving into S as a "success".

Put $U = \|p_{ij}\|_{i,j \in A}$. Let d be the maximal eigenvalue of a matrix U. We suppose that

(i) the chain has only one class C of essential states;

(ii) the class C does not contain cyclic subclasses;

(iii) $A \cap C \neq \varnothing$;

(iv) $0 < d < 1$.

Write

$$L(n) = \max\{k \leq n : \max_{0 \leq i \leq n-k} 1\{X_{i+1} \in A, \dots, X_{i+k} \in A\} = 1\}.$$

The random value $L(n)$ is the maximal length of series of "successes" in $X_1, \dots, X_n$.

There exists a large number of both articles and monographs dedicated to distribution of the random value $L(n)$ (see, e. g., references in [7]). In [8], there was proved the following

Theorem A. *The estimate $d_n = n^{-1/L(n)}$ for the maximal eigenvalue d is strictly consistent and asymptotically unbiased one.*

In the present Section we shall investigate the error of approximation of the value d by the statistics d_n.