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Step-by-step Solving of Ordered Interpolation Problem for Stieltjes Functions

Yu. M. Dyukarev* **and I. Yu. Serikova****

V. N. Karazin Kharkiv National University
pl. Svobody 4, Kharkiv, 61022 Ukraine

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Abstract—We investigate a step-by-step solving of ordered generalized interpolation problems for Stieltjes matrix functions and obtain a multiplicative representation for the sequence of resolvent matrices. The matrix factors in multiplicative representations of the resolvent matrices are expressed through the Schur–Stieltjes parameters, for which we obtain explicit formulas and give an algorithm of step-by-step solving of Stieltjes type interpolation problems. As examples, we consider step-by-step solutions of the Stieltjes matrix moment problem and the problems by Nevanlinna–Pick and Caratheodory.

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INTRODUCTION

In [1, 2] Schur first suggested an algorithm of step-by-step solving of the problem which is now known as the Schur problem. Many authors generalized the results of these papers for other classes of functions, for the matrix and operator cases. In the late sixties of the last century Potapov [3–6] proposed a new approach to the Schur analysis. For the matrix Schur problem, such an approach is described in detail in [7]. Monograph [8] contains analysis of the modern state of the theory of interpolation problems for the Nevanlinna functions and their analogs.

In [9–15] Potapov's approach was generalized to interpolation Stieltjes type problems. In these papers there were solved some typical Stieltjes type interpolation problems and suggested general schemes of solving such problems. But step-by-step solving of Stieltjes type interpolation problems remained unexplored. In this paper we investigate a step-by-step solving of ordered generalized Stieltjes type interpolation problems. General constructions are shown by examples of the Nevanlinna–Pick and Caratheodory problems, and the Stieltjes moment problem.

Now we introduce the basic definitions and give some necessary results. Let $\mathcal{G}_1$ and $\mathcal{G}_2$ denote separable Hilbert spaces, and let $\mathcal{H}$ be a finite-dimensional Hilbert space. We will assume that the scalar products in the Hilbert spaces are linear by the second argument. Let $\{\mathcal{G}_1, \mathcal{G}_2\}$ denote the set of all bounded linear operators acting from $\mathcal{G}_1$ to $\mathcal{G}_2$. Let $\{\mathcal{G}_1\}$ denote the set $\{\mathcal{G}_1, \mathcal{G}_1\}$, and let $\{\mathcal{G}_1\}_H$ be the set of all bounded Hermite operators in $\mathcal{G}_1$. An operator $A \in \{\mathcal{G}_1\}_H$ is called nonnegative if $(f, Af) \geq 0 \ \ \forall f \in \mathcal{G}_1$. We will denote by $\{\mathcal{G}_1\}_\geq$ the set of all nonnegative operators in $\mathcal{G}_1$. A nonnegative operator $A \in \{\mathcal{G}_1\}_\geq$ is called positive if it is invertible and $A^{-1} \in \{\mathcal{G}_1\}$. We will denote by $\{\mathcal{G}_1\}_>$ the set of all positive operators in $\mathcal{G}_1$. Let $A, B \in \{\mathcal{G}_1\}_H$. The inequality $A \geq B$ ($A > B$) means that $A - B \in \{\mathcal{G}_1\}_\geq$ (respectively, $A - B \in \{\mathcal{G}_1\}_>$). We denote by $I_{\mathcal{G}_1}$ and $O_{\mathcal{G}_1}$ the identity and zero operators in the Hilbert space $\mathcal{G}_1$. We denote by $O_{\mathcal{G}_1\mathcal{G}_2}$ the zero operator acting from $\mathcal{G}_1$ to $\mathcal{G}_2$. We will often omit the lower indices in notation of the zero and identity operators when this does not cause an ambiguity. If $f(z)$ denotes an operator function (OF) $f : \Omega \subset \mathbb{C} \to \mathcal{G}_1$, then the notation $f^*(z)$ is a

*E-mail: yu.dyukarev@karazin.ua.
**E-mail: irina.serikova@karazin.ua.