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THE SOLUTION OF THE PROBLEM OF $\mathbb{R}$-LINEAR CONJUGATION FOR THE CASE OF A HYPERBOLIC BOUNDARY LINE OF HETEROGENEOUS PHASES

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1. Problem definition

In this paper, we consider one well-known model of the theory of heterogeneous media. This classical model can be described as follows. One has to construct a plane-parallel stationary force field $\mathbf{v}(x,y) = (v_x, v_y) = \mathbf{v}_p(x,y)$, $(x,y) \in S_p$, $p = 1,2$, which is potential and solenoidal at each isotropic phase S_p of the two-phase medium under consideration:

$$\operatorname{div} \mathbf{v}_p(x,y) = 0, \quad \operatorname{rot} \mathbf{v}_p(x,y) = 0. \tag{1}$$

We assume that everywhere on the piecewise-smooth contact boundary ($\mathcal{L} = \partial S_1 \cap \partial S_2 \setminus T$) of heterogeneous phases S_1 and S_2, except the angular points T, the normal (tangent) components of the limiting values of vectors $\mathbf{v}_1$, $\mathbf{v}_2$ ($\widehat{\rho}_1\mathbf{v}_1$, $\widehat{\rho}_2\mathbf{v}_2$):

$$[\mathbf{v}_1(x,y)]_n = [\mathbf{v}_2 q(x,y)]_n, \quad [\widehat{\rho}_1\mathbf{v}_1(x,y)]_\tau = [\widehat{\rho}_2\mathbf{v}_2(x,y)]_\tau, \quad (x,y) \in \mathcal{L} \tag{2}$$

are equal. At the points of the set T, components of the vector $\mathbf{v}$ may have integrable singularities. In the second refraction condition (2), $\widehat{\rho}_p$ is the coefficient, constant for the phase S_p, which characterizes the physical features of the media. In implementations of concrete physical models, this coefficient usually takes scalar real nonnegative values. However, in some cases, for example, in the problems of electrodynamics, where electric fields are calculated taking into account the influence of a homogenous electromagnetic field (which is orthogonal to the current flow plane), the coefficient $\widehat{\rho}_p$ may be a tensor:

$$\widehat{\rho}_p = \rho_p \begin{pmatrix} 1 & \beta_p \\ -\beta_p & 1 \end{pmatrix}, \tag{3}$$

where $\rho_p \geq 0$ is the coefficient of resistance and $\beta_p \in \mathbb{R}$ is the Hall parameter of the phase material S_p, $p = 1,2$.

Further, we understand the physical plane (x,y) as a plane of complex variable $z = x + \mathrm{i}\, y$, and the vector $\mathbf{v}$ as a complex valued function $\mathbf{v}(z) = v_x + \mathrm{i}\, v_y$ of the complex argument $z = x + \mathrm{i}\, y$. In this case, tensor (3) can be identified with the complex number

$$\widehat{\rho}_p = \rho_p(1 - \mathrm{i}\,\beta_p). \tag{4}$$

Due to conditions (1), the complex-conjugate to $\mathbf{v}(z)$ function $v(z) = v_p(z) = v_{px}(x,y) - iv_{py}(x,y)$ is holomorphic in each component S_p. In the closure $\overline{S}_p$, the function $v(z)$ is continuous everywhere,

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