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Order-Optimal Methods for the Approximation of a Piecewise-Continuous Solution to a Certain Inverse Problem

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Abstract—The authors propose an order-optimal method for the approximation of a piecewise-continuous solution to a certain inverse problem and obtain exact by order bounds for the error of this approximation in the uniform metrics on segments of continuity of the exact solution to the problem.

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The experimental investigations of non-stationary processes of the heat-mass transfer in the interaction of fluid and gas flows with solids necessitate the definition of the heat-flux density towards the solids. In many cases the only way to find this value as a time function consists in solving the inverse problems of heat conductivity, measuring the temperature within the solid. Since these problems are very unstable, it is necessary to construct the maximally accurate solution methods for them and to estimate the inaccuracy of the desired values induced by the metering errors. Solving these problems (see [1], p. 23), one often deals with the interaction of the compression shocks with the surface of the solid. That is why the desired value in the inverse problem is considered as a discontinuous function. In papers [2]–[4] approximate solutions to the operator equation are built, using the space $V[a,b]$ of functions with bounded variation; they uniformly converge to the exact solution on the segments of its continuity.

In this paper we propose another approach which enables us to estimate the rate of the uniform convergence of approximate solutions on the segments of continuity of the exact solution. On the base of this approach we develop an order-optimal solution method for one inverse problem of temperature diagnostics [1].

1. BASIC NOTIONS AND DEFINITIONS

Let U, V, and Y be Banach spaces, let A and B be injective linear bounded operators, mapping the space U into Y and V into U, correspondingly. We denote by A^* and B^* the operators conjugated to A and B, we do by $M_{\overline{r}}$ the set $B\overline{S}_{\overline{r}}$, where $\overline{S}_{\overline{r}} = \{v : v \in V, \|v\| \le \overline{r}\}$.

Consider the following operator equation of the first kind:

$$Au = f, \quad u \in U, \ f \in Y; \tag{1}$$

we denote by $N_{\overline{r}}$ the set $AM_{\overline{r}}$, and we do by $A^{-1}_{N_{\overline{r}}}$ the restriction of the operator A^{-1} onto $N_{\overline{r}}$.

Definition 1. A set $M_{\overline{r}}$ is said to be the class of correctness for equation (1), if the operator $A^{-1}_{N_{\overline{r}}}$ is uniformly continuous.

One can easily prove the following assertion.

Theorem 1. *A set $M_{\overline{r}}$ is the class of correctness for equation* (1)*, if and only if the operator $A^{-1}_{N_{\overline{r}}}$ is continuous at the zero point.*

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