

UNIVERSAL VERSION OF GENERALIZED INTEGRAL CONSTRAINTS IN THE CLASS OF FINITELY ADDITIVE MEASURES, II

A.G. Chentsov

4. Relaxations of integral constraints under the condition that functionals in the basic system of conditions are step-wise

In the present article we continue the investigation started in [1]. We keep the same notation and terminology.

Consider another TS with a “unit” \mathbb{R}^Γ , namely, the TS

$$(\mathbb{R}^\Gamma, \otimes^\Gamma(\tau_\partial)) \quad (4.1)$$

corresponds to agreements of Section 2. To TS (4.1) one can relate a natural basis. If $f \in \mathbb{R}^\Gamma$ and $K \in \text{Fin}(\Gamma)$, then we denote by $T_0(f, K)$ the set of all functionals $g \in \mathbb{R}^\Gamma$ such that $\forall \gamma \in K : f(\gamma) = g(\gamma)$. Then, as one can easily see, $\forall f \in \mathbb{R}^\Gamma \forall K \in \text{Fin}(\Gamma) \forall g \in T_0(f, K) : T_0(f, K) = T_0(g, K)$. The family \mathcal{T}_0 of all sets $T_0(f, K)$, $(f, K) \in \mathbb{R}^\Gamma \times \text{Fin}(\Gamma)$, is a basis in TS (4.1). The proper topology $\otimes^\Gamma(\tau_\partial)$ is a family of all sets G , $G \subset \mathbb{R}^\Gamma$, such that for each of them we have

$$\forall f \in G \exists K \in \text{Fin}(\Gamma) : T_0(f, K) \subset G.$$

Consequently, for $f \in \mathbb{R}^\Gamma$, we have a fundamental system of neighborhoods f in TS (4.1): $\{T_0(f, K) : K \in \text{Fin}(\Gamma)\}$, which is a subfamily of \mathcal{T}_0 . To this circumstance the natural concept of converging directed sets in TS (4.1) is related. Indeed, if (\mathbb{H}, \preceq, h) is a directed set in \mathbb{R}^Γ and $v \in \mathbb{R}^\Gamma$, then the equivalence is valid

$$((\mathbb{H}, \preceq, h) \xrightarrow{\otimes^\Gamma(\tau_\partial)} v) \iff (\forall \gamma \in \Gamma \exists \alpha \in \mathbb{H} \forall \beta \in \mathbb{H} : (\alpha \preceq \beta) \implies (h(\beta)(\gamma) = v(\gamma))). \quad (4.2)$$

Since $\tau_{\mathbb{R}} \subset \tau_\partial$, for both TS (3.4) and (4.1) the relation holds

$$\otimes^\Gamma(\tau_{\mathbb{R}}) \subset \otimes^\Gamma(\tau_\partial), \quad (4.3)$$

which will be used in what follows. Note that the family \mathcal{Y}_∂ of all neighborhoods (see [2], p. 19) of a set Y in TS (4.1) possesses in view of (4.3) an intrinsic property of estimation $\mathcal{Y} \subset \mathcal{Y}_\partial$ with respect to the family \mathcal{Y} in Section 3. As in (3.15), we introduce a special class of neighborhoods of Y

$$\mathbb{N}_\partial(P) \triangleq \bigcup_{y \in Y} T_0(y, P) \in \mathcal{Y}_\partial \quad (4.4)$$

if $P \in \text{Fin}(\Gamma)$. One can easily see that

$$\widehat{\mathfrak{A}}_\partial \triangleq \{(\text{Adm})[H; Q] : (H, Q) \in \mathcal{Y}_\partial \times \text{Fin}(\Omega)\} \in \mathcal{B}(B_0(E, \mathcal{L})), \quad (4.5)$$

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