

ON COEFFICIENTS OF EXPANSION IN EIGENFUNCTIONS OF
A BOUNDARY VALUE PROBLEM WITH SPECTRAL PARAMETER
IN THE BOUNDARY CONDITIONS

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1. Introduction

The problem on determination of the magnitude of a current $i(x, t)$ in a conductor whose ends are grounded through a condensed resistors has the form

$$\begin{aligned} i_{xx} &= CLi_{tt} + (CR + GL)i_t + GRI, \quad i_x(0, t) - CR_0^{(1)}i_t(0, t) - GR_0^{(1)}i(0, t) = 0, \\ i_x(l, t) - CR_0^{(2)}i_t(l, t) - GR_0^{(2)}i(l, t) &= 0, \quad i(x, 0) = \varphi(x), \\ i_x(x, 0) &= (-R\varphi(x) - f'(x))/L \end{aligned}$$

(see [1], pp. 21, 176). Here $\varphi(x)$ and $f(x)$ are the magnitude of a current and its tension at the initial moment of time.

The problem under consideration admits a separation of variables with respect to time, therefore to solve it the Fourier method can be applied. In addition, a parameter arises in the corresponding problem in the boundary value conditions, which results in a violation of the minimality of chains of eigenfunctions in the space $L_2 \times L_2$ (see [2], p. 191). The substantiation of the expandability in series in chains of eigenfunctions for problems of this genus appeared in relatively recent time (see [2], pp. 190–229). It is carried out in spaces with more complicated structure. The problem of evaluation of coefficients of the corresponding series is not a trivial one. The coefficients of similar series were found only for some spectral problems (see [3], pp. 137–139) by means of the conjugate operator.

Instead of the mentioned concrete problem, in what follows we shall consider a general second order boundary value problem and determine for it the expansion coefficients for a pair of arbitrary functions (belonging to special spaces, see in [2]) in the series in eigenfunctions. We shall do it in two ways: the first one generalizes [3] and uses an explicit construction of the conjugate operator to A.A. Shkalikov's linearizer of problem (1)–(3), while the second one uses the possibility of representation of the problem in the form of a bundle of unbounded operators with further application of biorthogonality relations in [4], known for the operator bundles.

For the convenience of the reader, let us list the notation. We denote by \mathbb{C} the space of complex numbers; L_2 stands for the space of square summable functions given on the segment $[0, 1]$; W_2^1 means the Sobolev space of functions on the segment $[0, 1]$, \mathcal{W}_2^0 is the space $W_2^1 \times L_2$, \mathfrak{H} stands for the space $(L_2 \times \mathbb{C}^2)^2$. We denote by $(,)$, $[,]$, \langle , \rangle , and $(,)_{\mathfrak{H}}$ the scalar products in the spaces L_2 , $L_2 \times \mathbb{C}^2$, \mathcal{W}_2^0 , and \mathfrak{H} respectively, while y , \mathbf{y} , $\tilde{\mathbf{v}}$, and $\hat{\mathbf{y}}$ will mean the elements of the respective spaces.

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