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THE SOLUTION OF SADDLE POINT PROBLEMS BY METHODS WHICH USE MODEL SADDLE OPERATORS AT THE UPPER LEVEL

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Introduction

Let U and P be Euclidean spaces of N_u- and N_p-dimensional vectors, respectively. Consider the following nondegenerate real system of linear algebraic equations

$$L_0 z \equiv \begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} f \\ \varphi \end{pmatrix} \equiv F, \tag{1}$$

where $z = \{u, p\}$ is the vector of unknowns, $A = A^T > 0$ is a square $N_u \times N_u$-dimensional matrix, and B is a rectangular, in general case, matrix of dimension $N_u \times N_p$. Problems of this type arise in the numerical solution of systems of linear equations in the elasticity theory and in hydrodynamics (the Stokes problem) when mixed approximations for elliptic equations are used, and, also, when non-matching meshes are applied for solving boundary value problems (see, e. g., [1]–[3] and references there).

In this paper, we analyze the convergence of the following iterative method for solving problem (1)

$$\widetilde{L}\frac{z^{k+1} - z^k}{\tau_{k+1}} + L_0 z^k = F. \tag{2}$$

The superscript denotes the iteration counter. The matrix $\widetilde{L}$ has the following structure

$$\widetilde{L} = \begin{pmatrix} Q & B \\ B^T & -\alpha C, \end{pmatrix}$$

where $Q = Q^T > 0$, $C = C^T > 0$, α is a constant parameter of a preconditioner and τ_k is a variable iterative parameter. Further, the constants in the following matrix inequalities are supposed to be known:

$$\begin{aligned} \gamma C \le B^T A^{-1} B \le \Gamma C, &\quad 0 < \gamma < \Gamma, \\ \delta Q \le A \le \Delta Q, &\quad 0 < \delta \le \Delta. \end{aligned}$$

If necessary, we will scale these constants. For example, in order to simplify some considerations we will set the value δ (or Δ) to unit.

For the problems mentioned above, the algorithms with model saddle operators at the upper level were investigated in [4]–[8]. The closest formulation of a problem was considered in [4], [5]. The distinctive features of the present paper are the optimization of the spectrum of the preconditioned

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