ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 9, pp. 30–35. © Allerton Press, Inc., 2010.
Original Russian Text © N.A. Koreshkov, 2010, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2010, No. 9, pp. 36–42.

Finite-Dimensional Homogeneously Simple Algebras of Associative Type

N. A. Koreshkov[1*]

[1] *Kazan State University, ul. Kremlyovskaya 18, Kazan, 420008 Russia*

Received November 12, 2008

Abstract—In this paper, we describe finite-dimensional homogeneously simple algebras of associative type whose 1-component is a full matrix algebra. In addition, we prove that a finite-dimensional division ring of associative type over an algebraically closed field is isomorphic to a group algebra.

DOI: 10.3103/S1066369X10090033

Key words and phrases: *homogeneously simple algebra of associative type, division ring of associative type.*

Algebras of associative type discussed in this paper are partial case of algebras of Lie type introduced in [1, 2]. Properties of finite-dimensional algebras of associative type were studied in [3–5]. Note that many properties of these algebras are similar to the properties of finite-dimensional associative algebras.

In this paper, we study the structure of finite-dimensional homogeneously simple algebras of associative type. In [5], it has been proved that the component A_e (e is the identity of the grading group) of a homogeneously simple algebra of associative type over a field of zero characteristic is a semisimple associative algebra. If the basic field k is algebraically closed, then the component A_e is the direct sum of a number of full matrix algebras. This fact also takes place in the case of homogeneously simple algebras of associative type. For this reason, studying a homogeneously simple algebra of associative type, it is natural to consider first the case when the component A_e is a full matrix algebra. In the present paper, we consider this case without assumption that the basic field is algebraically closed. The result obtained is similar to the corresponding result for finite-dimensional homogeneously simple associative algebras (see [6]).

Recall the basic definitions. Let G be a multiplicative group with identity element e. Then an algebra $A = \bigoplus\limits_{g\in G} A_g$ over a field k is called a G-graded algebra (or a graded algebra with fixed group G) if $A_gA_h \subseteq A_{gh}$ for all $g, h \in G$. The subspaces A_g are called homogeneous components of the grading. In particular, A_e is called the identity component. For any $g \in G$, an element $a \in A_g$ is called a homogeneous element.

It is not necessary that all homogeneous components A_g of a G-graded algebra $A = \bigoplus\limits_{g\in G} A_g$ are nonzero. The set $\operatorname{supp} A = \{g \in G \mid A_g \neq 0\}$ is called the support of the grading.

A G-graded algebra $A = \bigoplus\limits_{g\in G} A_g$ is called an algebra of associative type if, for any collection $g_1, g_2, g_3 \in G$, there exists $\lambda = \lambda(g_1, g_2, g_3) \in k^* = k \setminus \{0\}$ such that

$$(a_{g_1}a_{g_2})a_{g_3} = \lambda a_{g_1}(a_{g_2}a_{g_3})$$

for any homogeneous elements $a_{g_i} \in A_{g_i}$, $i = 1, 2, 3$.

A subspace $V \subseteq A$ is called a homogeneous subspace if $V = \bigoplus\limits_{g\in G} (V \cap A_g)$. A subalgebra B in an algebra of associative type A is called homogeneous if it is a homogeneous subspace. A homogeneous (left, right, two-sided) ideal is defined in a similar manner.

*E-mail: Nikolai.Koreshkov@ksu.ru.