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THE INVARIANCE PRINCIPLE FOR SUMS OF INDEPENDENT RANDOM VALUES WITH REPLACEMENTS AND MODELS OF A FINANCIAL MARKET

Z.A. Enikeeva

In this paper, we prove the convergence of random polygonal lines to the Wiener process with the altered time in the space of continuous bounded functions on the real half-line. The random polygonal lines are defined by the sums of independent random values, where some random values are at random replaced with others. The replacements are defined by multiplication by the values of indicators defined on another probabilistic space, whose elements are considered as random parameters, and the convergence of the random polygonal lines is proved for almost all (a. a.) values of this random parameter. The results of this paper generalize the corresponding theorems from [1] and [2] which were obtained for sums of identically distributed random variables. Here we consider the more general schema of series of differently distributed random variables which satisfy the Lindeberg condition.

Note that the convergence of random polygonal lines is studied in many papers (see, e. g., [3]–[5]), as well as in this one.

The obtained limit theorems are used in several models of the stochastic financial mathematics; in particular, we adduce an analogue of the Black–Scholes formula of the fair price of the European option of a buyer.

1. Limit theorems

Let $\{\Omega_1, \mathfrak{A}_1, \mathbf{P}_1\}$ be a probabilistic space, $\mathbf{E}_1$ be the probabilistic average with respect to the probability $\mathbf{P}_1$, and let $A_{ji}(n) \in \mathfrak{A}_1$ be independent events for any $n \in \mathbf{N}$. Assume that the probabilities of these events do not depend on the index i and $\mathbf{P}_1(A_{ji}(n)) = a_{jn}$ for any $i \in \mathbf{N}$. By $\mathbb{I}_{ji}(n)(\omega_1)$ we denote indicators of events $A_{ji}(n)$, by $[b]$ and $\{b\}$ we do the integer and fractional parts of the number b, respectively.

Let $t \in \mathbb{R}_+$. Define a sequence of functions as follows: $f_n(t) = 1$, if $[k_n t] = 0$ and $f_n(t) = \mathbf{E}_1\, \mathbb{I}_{[k_n t]i}(n)\mathbb{I}_{([k_n t]-1)i}(n) \dots \mathbb{I}_{1i}(n) = a_{[k_n t]n} \dots a_{1n}$, if $[k_n t] > 0$.

Let the events $A_{ij}(n)$ fulfill the condition

(1) a function $f(t) \in C_b(\mathbb{R}_+)$ exists such that $f(t) = \lim\limits_{n\to\infty} f_n(t)$ for any $t \in \mathbb{R}_+$, where $C_b(\mathbb{R}_+)$ is a space of continuous bounded functions defined on $\mathbb{R}_+$.

Let $\{\Omega, \mathfrak{A}, \mathbf{P}\}$ be another probabilistic space. By $\mathbf{E}$ we denote the probabilistic average with respect to the probability $\mathbf{P}$.

Let $k_n \in \mathbf{N}$ be such that $k_n < k_{n+1}$ for any $n \in \mathbf{N}$ and $l \in \{1, 2\}$. Let $Y_{ni}^{(l)}$, $1 \le i \le k_n$, be sequences of series of independent in each series random values defined on $(\Omega, A, \mathbf{P})$. Assume that for any $n \in \mathbf{N}$ the families $\{Y_{ni}^{(1)}, 1 \le i \le k_n\}$ and $\{Y_{ni}^{(2)}, 1 \le i \le k_n\}$ are independent. Let for $l \in \{1, 2\}$ the following conditions hold: