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Almost Contact Kähler Manifolds of Constant Holomorphic Sectional Curvature

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Abstract—We introduce the notion of an almost contact Kähler structure. We also define the holomorphic sectional curvature of the distribution of an almost contact Kähler structure with respect to an interior metric connection and establish relations between the φ-sectional curvature of an almost contact Kähler manifold and the holomorphic sectional curvature of the distribution of an almost contact Kähler structure.

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1. INTRODUCTION

Let $(\varphi, \vec{\xi}, \eta, g)$ be an almost contact metric structure on a manifold X. There are at least two geometries for discussion in this situation: The geometry of the nonholonomic manifold (X, D, g) and the geometry of the almost contact metric space $(\varphi, \vec{\xi}, \eta, g, X, D)$. One can introduce criteria which will allow one to compare these geometries. Obviously, one can expect that these geometries have more similar properties in the case of a Sasakian structure. In [1] on a contact metric space X, in addition to the Levi-Civita connection $\widetilde{\nabla}$, they a linear connection ∇, called the D-connection, was introduced. In the general case, this connection is not a metric one, and the following statement holds: A contact metric space is a Sasakian space if and only if $\nabla\varphi = 0$ ([1], p. 1963). In the same paper, the Kähler contact distribution was introduced as well as the notion of holomorphic sectional curvature of the Kähler contact distribution with respect to the connection ∇. It was proved that the Kähler contact distribution is a distribution of constant holomorphic sectional curvature with respect to ∇ if and only if X is a Sasakian space form. In conclusion the author writes ([1], P. 1967): "…we can say that the study of the contact distribution (D, φ, g) with the use of the D-connection is an alternative to the study of the contact metric manifold with the Levi-Civita connection". In the present paper, we first study more general almost contact metric spaces than Sasakian ones, namely, almost contact Kähler spaces (AKCS) in which the equalities $\omega(\varphi\vec{x}, \varphi\vec{y}) = \omega(\vec{x}, \vec{y})$, $\Omega = d\eta$ are not supposed to be held. Second, along with the geometry of an ACKS, we study the geometry of the corresponding nonholonomic manifold. Such an approach to the study of almost contact metric structures allows us, on one hand, to use the techniques of nonholonomic geometry for obtaining new results and, on the other hand, gives a new understanding of already known results in the geometry of almost contact metric spaces.

By definition, an almost contact metric structure is Sasakian if it is normal, i.e., $N_\varphi + 2d\eta \otimes \vec{\xi} = 0$, where N_φ is the Nijenhuis torsion for the tensor φ, and the equality $\Omega = d\eta$ holds, where $\Omega(\vec{x}, \vec{y}) = g(\vec{x}, \varphi\vec{y})$ is the fundamental form of the structure. Rejecting the condition $\Omega = d\eta$ and replacing the condition $N_\varphi + 2d\eta \otimes \vec{\xi} = 0$ by the weaker condition $N_\varphi + 2(d\eta \circ \varphi) \otimes \vec{\xi} = 0$, one obtains a space with almost contact Hermitian structure. If the fundamental form of an almost contact Hermitian space is

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