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INDUCED CONNECTIONS ON SUBMANIFOLDS IN SPACES WITH FUNDAMENTAL GROUPS

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Introduction

The theory of congruences and pseudocongruences of subspaces of a projective space is closely related to the theory of varieties with degenerate Gauss maps.

In a three-dimensional projective space $\mathbb{P}^3$ as well as in three-dimensional spaces endowed with a projective structure (such as affine, Euclidean, and non-Euclidean spaces), the theory of congruences was studied by many geometers. Extensive monographs on this subject were published (see, e. g., [1]).

In this paper, we establish a relation of the theory of varieties with degenerate Gauss maps in projective spaces with the theory of congruences and pseudocongruences of subspaces and show how these two theories can be applied to the construction of induced connections on submanifolds of projective spaces and other spaces endowed with a projective structure.

1. Basic equations of a variety with a degenerate Gauss map

A smooth n-dimensional variety X of a projective space $\mathbb{P}^N$, $X \subset \mathbb{P}^N$, is called *a tangentially degenerate variety* or *a variety with a degenerate Gauss map* if the rank of its Gauss map

$$\gamma : X \to \mathbb{G}(n, N)$$

is less than n, $0 \leq r = \operatorname{rank} \gamma < n$. Here $\gamma(x) = T_x(X)$, and $T_x(X)$ is the tangent subspace to X at x considered as an n-dimensional projective space $\mathbb{P}^n$. The number r is also called the *rank* of X, $r = \operatorname{rank} X$. The case $r = 0$ is a trivial one: it gives just an n-plane.

Let $X \subset \mathbb{P}^N$ be an almost everywhere smooth n-dimensional variety with a degenerate Gauss map. Suppose that $0 < \operatorname{rank} \gamma = r < n$. Denote by L a leaf of the map

$$\gamma : L = \gamma^{-1}(T_x) \subset X; \quad \dim L = n - r = l.$$

As was proved in [2] (see theorem 3.1, p. 95), a variety with a degenerate Gauss map of rank r foliates into its plane leaves L of dimension l along which the tangent subspace $T_x(X)$ is fixed. The tangent subspace $T_x(X)$ is fixed when a point x moves along regular points of L. This is the reason that we denote it by T_L, $L \subset T_L$. A pair (L, T_L) on X depends on r parameters.

The foliation on X defined as indicated above is called the *Monge–Ampère foliation.*

The varieties of rank $r < n$ are multidimensional analogues of developable surfaces of a three-dimensional Euclidean space.

The main results on the geometry of varieties with degenerate Gauss maps and further references can be found in [3] (Chap. 4) and in [2].